Sticky Prices, Endogenous Export Participation, and Real Exchange Rate Fluctuations*

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Abstract

This paper develops a two-country DSGE model where firms optimally decide whether to engage in the export market. The heterogeneous firms face infrequent opportunities to adjust prices and are subject to fixed costs of exporting drawn independently from a time-invariant distribution. This endogenous firms' exporting decision provides a new additional channel for the transmission of monetary shocks. The model is able to generate several important features of the firm-level data on international trades as documented by Bernard and Jensen (2005). A novel feature of the model is that consistent with the data evidence, some low-productivity firms may still find it profitable to export even though they are less likely to export on average. The model predicts that under financial autarky and balanced trade and when the economy is dominated by monetary shocks, endogenous export participation magnifies the real exchange rate volatility when producer-currency pricing is assumed. Under local-currency pricing, the volatility is dampened. The contrasting response of the changes in relative availability of goods varieties across countries (extensive margin) following monetary shocks provides the exhibited result. Variations in the relative set of available varieties serve to modify the extent of expenditure switching, which importantly affect the real exchange rate fluctuations.

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1 Introduction

The traditional explanation for the observed high real exchange rate fluctuations in the data is that they are generated by the interaction between sticky goods prices and monetary shocks. The nature of the goods in the market also matters for the behavior of the real exchange rate and its deviations from purchasing power parity (PPP). This large and persistent departure from PPP is usually assumed to derive either from the presence of preset non-traded goods or from the deviations of the law of one price of traded goods when all goods are assumed to be traded internationally. However, several recent evidence on international trade data suggests that a reconsideration is appropriate. First, Bernard and Jensen (2004) document that there are large movements of firms entering and exiting the export market - specifically, about 13% of firms appear to switch their export status every year. This evidence means that even among tradable goods, some of them may become non traded in any given period. Second, there is now substantial and consistent evidence for many countries that firms who export are in the minority, there is a substantial heterogeneity in firms' productivity level, and firms who are more productive and have larger size are more likely to export (e.g. Bernard and Jensen, 1999, 2004, Yan Aw, et. al., 2000, Das, et. al., 2001, and Bernard, et. al., 2005). Figure 1.A displays the relationship between firms' productivity and the probability of exporting in the data as documented by Bernard, Jensen, Eaton, and Kortum (2005 - henceforth BEJK).

Existing open economy models do not incorporate these features. There are, of course, many studies with sticky goods prices, but these models do not consider any heterogeneity in firms' productivity and do not allow for endogenous export participation by firms, e.g. Chari, et. al. (2002), Betts and Devereux (1996, 2000), Bergin and Feenstra (2001), and Kollmann (2001). There is also a growing literature that incorporate productivity heterogeneity and endogenous export participation, with the work of Ghironi and Melitz (2005) being particularly notable.[1] However, there are no models that include all three dimensions of sticky goods prices, heterogeneity in productivity,

[1] Other work in this literature includes Bergin and Glick (2003), who build a simple endowment model with heterogeneity in terms of firms productivity with endogenous tradeability assumption to explains the low volatility of nontraded and traded goods in conjunction with high real exchange rate volatility. Naknoi (2007) and Bergin, Glick, and Taylor (2006) also builds models with a continuum of monopolistically-competitive heterogeneous firms where firms can optimally choose whether to export.

and endogenous export participation and that explore the implications of these features for real exchange rate dynamics when the economy is dominated by monetary shocks.

This paper fills the gap by building a two-country dynamic general equilibrium model where firms optimally decide whether to engage in the export market. Each heterogenous firm produces a differentiated good (variety) and all differentiated goods are assumed to be tradable. Firms are also assumed to face infrequent opportunities to adjust prices in our model. Specifically, at the start of every period, firms know whether they are allowed to adjust prices. Firms have no power over the timing of their price adjustment decision. Besides this infrequent price adjustment assumption, the heterogenous firms also face a fixed cost of exporting, drawn every period from a time-invariant distribution. Firms then decide whether to enter the export market given the prices of their goods and the fixed cost draw. The assumptions of nominal rigidity and the fixed cost of exporting serve to provide an endogenous channel of international price discrimination that leads to the deviation of the law of one price. Moreover, this endogenous firms' exporting decision provides another channel for the transmission of monetary shocks.

The model is able to generate several important features of the firm-level data on international trades described above. First, the assumption that each firm must pay a fixed cost of exporting and the fact that each firm can optimally choose whether to export lead to some activities of firms entering and exiting the export market. Second, heterogeneous productivity levels and random fixed costs mean that the probability of exporting is increasing with productivity and exporting firms are on average more productive and have larger size. A novel feature of the model is that consistent with the data evidence in figure 1.A, some low-productivity firms may still find it profitable to export even though they are less likely to export on average. This result is not possible without the heterogenous random fixed assumption in addition to heterogenous firms' productivity levels.

Within the framework of the model, we investigate the role of endogenous export participation in generating real exchange rate fluctuations when the economy is dominated by monetary shocks. We first derive an accounting equation involving the evolution of relative availability of goods varieties across countries that holds across different assumptions regarding the degree of financial market completeness and the currency-denomination of export goods' prices. These variations in the relative set of available varieties serve to modify the extent of expenditure-switching, which in turn affects the real exchange rate fluctuations. Under the assumption of financial autarky and balanced trade, the model predicts that endogenous export participation (tradeability) has an important impact on the real exchange responses following a monetary shock. However, this impact is sensitive to the assumption about the currency denomination of goods prices. When export prices are denominated in producers' currency, the real exchange rate fluctuations tend to magnify due to endogenous changes in the relative availability of goods varieties. Specifically, following a monetary expansion in the home country, the real exchange rate tend to depreciate by more since relative consumption across countries increases by more due to the decrease in the relative availability of goods varieties. On the contrary, under local-currency pricing assumption, the real exchange rate response is dampened. The contrasting responses of the relative availability of goods varieties (extensive margin) under the two currency-denomination assumptions serve to generate these two different results.

The rest of the paper is organized as follows. Section 2 describes the model under the benchmark producer-currency pricing case. Section 3 discusses the model solution technique and describes the economy at the steady state. Section 4 derives an accounting equation involving variations in the relative availability of varieties. We also discuss the implication of these variations for the real exchange rate fluctuations. Section 5 provides the impulse responses following a monetary expansion at the home country. The result on the alternative local-currency pricing assumption is also presented. Section 6 concludes.

2 The model

The world economy consists of two countries, home and foreign. In each country there are a representative infinitely-lived households, a continuum of firms with heterogenous productivity levels indexed on the unit interval, and a monetary authority. The central components of the model is the addition of nominal price rigidity and the assumption that each firm (plant) must pay a fixed cost of exporting drawn independently each period from a time-invariant distribution. Households derive utilities from consumption and leisure. The aggregate consumption in the home country is given by $c_t = \left(\int_{i \in \Omega_t} c_t(i)^{\theta - 1/\theta} \right)^{\theta/\theta - 1}$, where θ represents the elasticity of substitution across goods varieties. Hence, households are assumed to value varieties and each variety is valued equally. The demand for a variety i is then given by $c_t(i) = p_t(i)^{-\theta} c_t$, where $p_t(i)$ represents the real price of good i faced by consumers in their own currency.

At each period t only a subset $\Omega_t \subset \Omega$ is available for consumption due to the existence of fixed costs of exporting that prevent some foreign firms to export goods to the home country. Hence in any given period, all domestic goods within the unit measure are available for home consumption, but only a subset of imported goods within the unit measure is available for home consumption. The foreign country has an identical setup. In terms of the financial market assumption, we assume financial autarky for the moment to focus attention on the goods market. Hence, only intermediate goods are traded internationally.

For the rest of the paper, foreign country variables are represented by asterisks. We use superscripts D and X to represent domestic and export operations, respectively. By domestic (export) operation, we mean the location of the market where goods are sold.

2.1 Households' intertemporal choice

The representative household in the home country chooses consumption and work effort to maximize the lifetime utility function

$$U = \sum\nolimits_{t=0}^{\infty} \beta^t \left[\frac{1}{1-\sigma} c_t^{1-\sigma} - \chi \frac{1}{1+\eta} n_t^{1+\eta} \right]$$

where σ and η represent the inverse of elasticity of intertemporal substitution and the inverse of labor supply elasticity, respectively. The foreign counterpart has the same preference as well. In each period households receive income from their labor effort with nominal wage W_t, nominal one-period bond (B_t) from last period, and dividends from their ownership of home firms (D_t). Households also inherit their previous-period holding of portfolios of intermediate-goods firms (x_t). They then pick the amount of consumption for that period, buy current period bonds (B_{t+1}), and may buy more claims on the ownership of the intermediate goods firms. Hence, the nominal budget constraint is given by

$$P_t c_t + \frac{1}{1+R_t} B_{t+1} + x_{t+1}(V_t - D_t) = x_t V_t + B_t + W_t n_t$$

Note the nominal value of the portfolio of firms, V_t, is the pre-dividend value. Deflating by the consumption-based price index P_t, we have the real budget constraint

$$c_t + \frac{1}{1+R_t} b_{t+1} + x_{t+1}(v_t - d_t) = x_t v_t + \frac{1}{1+\pi_t} b_t + w_t n_t \tag{1}$$

where $1 + \pi_t = P_t / P_{t-1}$ represents the current-period gross inflation level.

Next let λ_t be the multiplier of the constraint. Taking the first order conditions of the households' problem and rearranging, we have

$$c_t^{-\sigma} = \lambda_t \tag{2}$$

$$\chi n_t^{\eta} = \lambda_t w_t \tag{3}$$

$$\lambda_t (v_t - d_t) = \beta E_t [\lambda_{t+1} v_{t+1}] \tag{4}$$

$$\lambda_t \frac{1}{1+R_t} = \beta E_t \left[\lambda_{t+1} \frac{1}{1+\pi_{t+1}} \right] \tag{5}$$

Equation 2 states that the marginal utility of consumption is equal to the cost of consuming. Equations 3 governs the labor supply. Equations 4 and 5 are the Euler equations for bonds and the portfolio of firms, respectively. Last, for now we assume that money demand is given by $M_t/P_t = c_t$.

2.2 The intermediate-good firms

Here we discuss the setup of the intermediate-good firms of the home country - the foreign country counterparts have similar setups. Each firm in the home country produces a differentiated good (variety) with varying degrees of relative productivity z and with a linear technology given by $y_t(z) = (Z_t z) n_t$ where Z_t is the aggregate productivity level. The firm-specific productivity level z is attached to each firm forever. The distribution of the relative productivity is Pareto with support $[z_{\min}, \infty]$ and CDF $G(z) = 1 - (z_{\min}/z)^k$. All of these differentiated goods are tradable but only some of them become traded due to the existence of the fixed cost of exporting. This decision whether to export is made each period. International trades are done by firms directly and firms are assumed to have monopoly powers so that they can engage in pricing to market. Labor is assumed to be immobile internationally.

We introduce the price rigidity by adopting the price adjustment setup used in Levin (1991) and Khan, King, and Wolman (2003) - as we will see later, this price adjustment setup can generate similar distribution of price duration as the evidence in Gopinath and Rigobon (2007). In this setup firms have infrequent opportunities to adjust prices and the (fixed) probability of price adjustment is independent of firms' previous prices and the time since last adjustment - hence, firms have no power over the timing of their price adjustment decision. Specifically, the probability of price adjustment for firms that last adjusted their prices j periods ago is given by α_j. This probability is increasing with j and the longest period of price fixity is J. This setup means that firms can be classified into various bins $j = 0, 1, ..., J - 1$, where each bin j comprises the firms that last adjust their prices j periods ago. The distribution of firms according to the time since last adjustment is then given by $\omega_j = (1 - \alpha_j)\omega_{j-1}$, for $j = 1, ..., J - 1$, and with $\sum_{j=0}^{J-1} \omega_j = 1$.[2] Since the mass of firms is unity, ω_j can also be interpreted as the proportion of firms (with varying degree of productivity levels) that last adjust their prices j periods ago. Furthermore, since the opportunity to adjust prices is independent of firms' relative productivity levels, each bin j comprises of firms with relative productivity distributed according to a Pareto distribution as well, with the same parameters as the whole distribution. We can then analyze each firm in each bin separately.

In addition to facing infrequent opportunities to adjust prices, each firm is also subject to a fixed cost of exporting, independently drawn from a time-invariant distribution. The timing of the firms' decision in terms of pricing and exporting can be thought as follows. At the beginning of each period, each firm knows whether it is allowed to adjust prices but does not know yet its fixed cost of exporting. Adjusting firms then pick both optimal domestic and export prices (taking into account that they may or may not export given the future draws of export fixed costs), while non-adjusting firms keep their prices from the previous period. Each firm then draws the fixed cost and decides whether to enter the export market given the fixed cost draw, the product's prices for domestic and export sales, and the aggregate macroeconomic condition.

The assumption that firms are subject to a fixed cost of exporting independently drawn from a distribution merits some explanations. First, on the per-period fixed cost instead of a sunk cost.

[2] This price adjustment process encompasses Taylor and Calvo pricing as a special case. When $J- > \infty$ and the probability of adjustment α_j is equal for all j, we have the familiar Calvo price setting. For any J and when each firm is only allowed to adjust prices every J period ($\alpha_J = 1$ and $\alpha_j = 0$ for $j = 1, ..., J - 1$), we have Taylor price setting.

Several studies, e.g. Bernard and Jensen (2004) and Das, Roberts, and Tybout (2001), have documented that entry costs are large and most of them are sunk upon entry. Bernard and Jensen (2004) documents that firms that export the previous period have 66% higher probability of exporting again in the current period compared to those not exporting previously, which is consistent with the sunk cost story. The per-period fixed cost assumed in this paper obviously cannot generate this observed feature of the data. We adopt the per-period fixed costs assumption to make the solution algorithm much simpler, but without leaving out several important features (described below) that we are interested in. Next, the assumption that the fixed export cost may be different (heterogenous) across firms and are drawn independently from a continuous distribution is unconventional to the standard assumption that the fixed cost is homogenous across firms (e.g. Melitz, 2003). In the model below, we choose to adopt heterogenous fixed costs random independently drawn from a distribution in order to generate a nice steady state solution under the sticky price environment assumed in the model. Without this uncertainty of the fixed cost draw, the model would exhibit discrete aggregate (exporting) behavior in the steady state. Moreover, the adoption of the heterogenous fixed costs would also generate an important feature of the data: higher productivity firms have a higher probability of exporting, but without limiting low-productivity firms to export. As documented Bernard, Eaton, Jensen, and Kortum (2005, henceforth BEJK) in figures 2A and 2B, some firms with a low productivity level also export, but with lower probability (proportion) than higher-productivity firms. The homogenous fixed cost assumption (e.g. Melitz, 2003 and Ghironi and Melitz, 2005) cannot generate this feature; this assumption results in a cutoff export productivity level in which only firms with productivity levels higher than this cutoff would export, while those firms below this cutoff would not. Heterogenous fixed costs is needed to generate this feature. Despite some limitations of the fixed cost assumption in this model, it is rich enough to generate some of novel features of international trade in the data: (i) probability of exporting is increasing with productivity level without excluding the possibility of some low-productivity firms exporting, (ii) on average, exporting firms are more productive and have larger size (higher market share) than non-exporting firms, (iii) there are activities of firms entering and exiting the export market.

2.2.1 Adjusting firms' problem

The domestic and foreign sales of firms are separable by construction. Below we consider these two branches for the home firms separately in terms of their optimal pricing policies. Foreign firms face similar problems. For both branches, each firm is identified by its productivity level z and the time since the last price adjustment j.

Domestic sales The domestic demand for a variety produced by a $j-z$ home firm (a firm with relative productivity z that last adjusted its price j periods ago) is given by $\left(\frac{P^D_{j,t}(z)}{P_t}\right)^{-\theta} c_t$, where $P^D_{j,t}(z)$ is the nominal price of its product for domestic sales. Given this demand schedule, the domestic price, and the real marginal cost $\frac{w_t}{Z_t z}$, the one-period domestic profit for this firm is given by

$$d^D_j(z) = \left[\left(\frac{P^D_{j,t}(z)}{P_t}\right)^{1-\theta} - \left(\frac{P^D_{j,t}(z)}{P_t}\right)^{-\theta} \frac{w_t}{Z_t z}\right] c_t$$

Each adjusting firm z chooses domestic price to maximize the discounted present value of its lifetime profits taking into account that it may not be able to adjust its price again until some future periods. Note that by adjusting firms, we mean all the firms that are now reside in bin $j = 0$. Letting s_t as the vector of state variables (to be defined later), the problem for an adjusting z firm can be defined recursively as

$$\begin{aligned} v^D_0(z; s_t) \quad &= \quad \max_{P^D_{0,t}(z)} d^D_{0,t}(z) + \beta E_t \frac{\lambda_{t+1}}{\lambda_t} \alpha_1 v^D_0(z; s_t) \\ &\quad + \beta E_t \frac{\lambda_{t+1}}{\lambda_t} (1-\alpha_1) v^D_1(z; s_{t+1}) \end{aligned} \tag{6}$$

where $v^D_0(z; s_t)$ is the domestic value of adjusting firm z at period t. The value functions for the non-adjusting firms ($j = 1, ..., J-1$) can similarly defined recursively as

$$\begin{aligned} v^D_j(z; s_t) \quad &= \quad d^D_{j,t}(z) + \beta E_t \frac{\lambda_{t+1}}{\lambda_t} \alpha_{j+1} v^D_0(z; s_t) \\ &\quad + \beta E_t \frac{\lambda_{t+1}}{\lambda_t} (1-\alpha_{j+1}) v^D_{j+1}(z; s_{t+1}) \end{aligned} \tag{7}$$

$$v^D_{J-1}(z; s_t) = d^D_{J-1,t}(z) + \beta E_t \frac{\lambda_{t+1}}{\lambda_t} v^D_0(z; s_t) \tag{8}$$

Note that equation 7 holds for $j = 1, ..., J-2$ and there is no max operator in the two equations above since these firms are not allowed to adjust prices in the current period. Furthermore, firms in bin $J-1$ know that they would be allowed to adjust prices next period with probability one as reflected in equation (8). Solving for the adjusting firms' problem recursively, the optimal domestic price for any adjusting z firm is given by

$$P_{0,t}^{D}(z)=\frac{\theta}{\theta-1}\frac{\sum_{j=0}^{J-1}\beta^{j}\frac{\omega_j}{\omega_0}E_t\frac{\lambda_{t+j}}{\lambda_t}\frac{w_{t+j}}{Z_{t+j}z}(P_{t+j})^{\theta}c_{t+j}}{\sum_{j=0}^{J-1}\beta^{j}\frac{\omega_j}{\omega_0}E_t\frac{\lambda_{t+j}}{\lambda_t}(P_{t+j})^{\theta-1}c_{t+j}}\ .$$

This pricing policy is similar to the expression under Calvo (1983) price setting in a sense that the optimal price is a function of future marginal costs since firms have to take into account the possibility of not being able to adjust prices in the future for some periods.[3]

It can be shown that we can solve the domestic branch of the firms' problem using a representative firm as in Ghironi and Melitz (2005). That is on the aggregate level, the domestic branch of the model is similar to the one in which all firms producing for the domestic market with the same productivity $\tilde{z}_j^D=\tilde{z}^D$ given by

$$\tilde{z}^{D}=\left[\int_{z_{\min}}^{\infty}z^{\theta-1}dG(z)\right]^{\frac{1}{\theta-1}} \tag{9}$$

Taking into account this structure, we can solve for various "averages" for the domestic branch. Let $\tilde{v}_{j,t}^{D}$ be the average domestic value of firms in bin j. Also define $\tilde{d}_{j,t}^{D}$ as the average domestic profits and $\tilde{P}_{j,t}^{D}$ as the consumption-based average domestic price for firms in bin j.[4] Using the special average productivity $\tilde{z}^D$, it can be shown that $\tilde{v}_j^D(s_t)=v_j^D(\tilde{z}^D;s_t)$, $\tilde{d}_{j,t}^{D}=d_{j,t}^{D}(\tilde{z}^D)$, and $\tilde{P}_{j,t}^{D}=P_{j,t}^{D}(\tilde{z}^D)$.[5] Hence, in solving for this average optimal domestic price, we can imagine as if each adjusting firm (irrespective to the relative productivity level) maximizes

$$\begin{aligned}\tilde{v}_0^D(s_t) &= \max_{\tilde{P}_{0,t}^{D}} \tilde{d}_{0,t}^{D}+\beta E_t\frac{\lambda_{t+1}}{\lambda_t}\alpha_1\tilde{v}_0^D(s_{t+1}) \\ &\quad +\beta E_t\frac{\lambda_{t+1}}{\lambda_t}(1-\alpha_1)\tilde{v}_1^D(s_{t+1})\end{aligned} \tag{10}$$

Next, the non-adjusting firms' average value recursion are given by

$$\begin{aligned}\tilde{v}_j^D(s_t) &= \tilde{d}_{j,t}^{D}+\beta E_t\frac{\lambda_{t+1}}{\lambda_t}\alpha_{j+1}\tilde{v}_0^D(s_{t+1}) \\ &\quad +\beta E_t\frac{\lambda_{t+1}}{\lambda_t}(1-\alpha_{j+1})\tilde{v}_{j+1}^D(s_{t+1}) \quad , j=1,...,J-2\end{aligned} \tag{11}$$

[3] For example, this expression is similar to the one in a close-economy model in King and Goodfriend (1997).

[4] Formally, $\tilde{v}_j^D(s_t)=\int_{z_{\min}}^{\infty}v_{j,t}^D(z;s_t)dG(z)$, $\tilde{d}_{j,t}^{D}=\int_{z_{\min}}^{\infty}d_{j,t}^{D}(z)dG(z)$, and $\tilde{P}_{j,t}^{D}=\left[\int_{z_{\min}}^{\infty}(P_{j,t}^{D}(z))^{1-\theta}dG(z)\right]^{1/(1-\theta)}$

[5] See Melitz (2003) for details.

$$\tilde{v}^D_{J-1}(s_t) = \tilde{d}^D_{J-1,t} + \beta E_t \frac{\lambda_{t+1}}{\lambda_t} \tilde{v}^D_0(s_{t+1}) \tag{12}$$

Solving for for this problem recursive we arrive at the average optimal domestic price in the form of

$$\tilde{P}^D_{0,t} = \frac{\theta}{\theta-1} \frac{\sum_{j=0}^{J-1} \beta^j \frac{\omega_j}{\omega_0} E_t \frac{\lambda_{t+j}}{\lambda_t} \frac{w_{t+j}}{Z_{t+j}\tilde{z}^D} (P_{t+j})^\theta c_{t+j}}{\sum_{j=0}^{J-1} \beta^j \frac{\omega_j}{\omega_0} E_t \frac{\lambda_{t+j}}{\lambda_t} (P_{t+j})^{\theta-1} c_{t+j}} \ . \tag{13}$$

Export sales The structure of the export branch is similar to the domestic branch except that adjusting firms have to also take into account the probability of not exporting in the current or future periods if they draw a high enough fixed cost of exporting. As we will see later, the optimal export price will depend on these expected probabilities of exporting. Below we describe the export pricing policy for an adjusting z firm.

Given the CES aggregation and nominal export price $P^X_{j,t}(z)$, the demand from foreign consumers for goods produced by any home $j - z$ firm is given by $[S_t^{-1} P^X_{j,t}(z)/P^*_t]^{-\theta} c^*_t$, where S_t, P^*_t, and c^*_t represent the nominal exchange rate (home-currency price of foreign currency), foreign CPI price level, and foreign aggregate consumption, respectively. Note that goods prices are denominated in exporters' currency (producer currency pricing - PCP). Given this export demand for its product, the export price for its product, the production technology, the real wage w_t, and the export fixed cost draw, the per-period real export profit for a typical $j - z$ firm is given by

$$d^X_{j,t}(z;\xi_t) = \begin{cases} \pi^X_{j,t}(z) - w_t\xi_t & \text{in the export market} \\ 0 & \text{out of the export market} \end{cases} \tag{14}$$

where $\pi^X_{j,t}(z) = \left[(P^X_{j,t}(z)/P_t)^{1-\theta} - (P^X_{j,t}(z)/P_t)^{-\theta} \tau_t \frac{w_t}{Z_t z} \right] Q^\theta c^*_t$ is the gross export profit. Firms that choose not to export do not have to pay the fixed cost and receives zero export profit. Here, $Q_t = S_t P^*_t / P_t$ is the real exchange rate and $\tau_t \geq 1$ is the iceberg export cost.

Since each adjusting z firm has to optimally pick an export price before the realization of the fixed cost draw, each firm does not know whether it would be profitable to export in the current period. For the purpose of deciding the optimal export price each adjusting firm then has to rely on its expected export profit, taking into account the current and future probabilities of exporting. Let $\alpha^X_{j,t}(z)$ (to be defined later) as this probability of exporting for a typical $j - z$ firm and let $F(.)$ be the cumulative probability distribution (CDF) of the fixed cost distribution. The expected export profit for a typical $j-z$ firm (prior to the fixed cost draw) is thus given by

$$\tilde{d}^X_{j,t}(z) = \alpha^X_{j,t}(z) \cdot \left[\pi^X_{j,t}(z) - w_t \frac{1}{\alpha^X_{j,t}(z)} \int_0^{F^{-1}(\alpha^X_{j,t}(z)} xf(x)dx \right]$$

where $\frac{1}{\alpha^X_{j,t}(z)} \int_0^{F^{-1}(\alpha^X_{j,t}(z)} xf(x)dx$ represents the average labor unit used for the purpose of entering the export market by exporting $j-z$ firms. Here $\alpha^X_{j,t}(z)$ can also be viewed as the proportion of $j-z$ firms serving the export market. Rearranging this expression, we have

$$\tilde{d}^X_{j,t}(z) = \alpha^X_{j,t}(z) \cdot \pi^X_{j,t}(z) - w_t \cdot \Xi_{j,t}(z) \tag{15}$$

where $\Xi_{j,t}(z)$ is the total labor associated with the fixed cost of exporting by $j-z$ firms given by

$$\Xi_{j,t}(z) = \int_0^{F^{-1}(\alpha^X_{j,t}(z))} xf(x)dx \tag{16}$$

Each adjusting firm z correctly predicts this expected export profit when choosing the optimal price prior to the fixed cost draw. Although each firm z solves the optimal problem independently we can use the fact that each adjusting z firm has an identical problem since the optimal price is chosen prior to the realization of the fixed cost draw. Thus, when solving for the optimal price, we can view as if each z firm maximizes the average export value of all z firms. Hence, if we let $\tilde{v}^X_j(z; s_t)$ be the average export value of $j-z$ firms, the adjusting z firms problem for the export branch is recursively given by

$$\begin{aligned} \tilde{v}^X_0(z; s_t) &= \max_{P^X_{0,t}(z)} \tilde{d}^X_{0,t}(z) + \beta E_t \frac{\lambda_{t+1}}{\lambda_t} \alpha_1 \tilde{v}^X_0(z; s_{t+1}) \\ &\quad + \beta E_t \frac{\lambda_{t+1}}{\lambda_t} (1-\alpha_1) \tilde{v}^X_1(z; s_{t+1}) \end{aligned} \tag{17}$$

For non-adjusting firms the value recursions are as follows

$$\begin{aligned} \tilde{v}^X_j(z; s_t) &= \tilde{d}^X_{j,t}(z) + \beta E_t \frac{\lambda_{t+1}}{\lambda_t} \alpha_{j+1} \tilde{v}^X_0(z; s_{t+1}) \\ &\quad + \beta E_t \frac{\lambda_{t+1}}{\lambda_t} (1-\alpha_{j+1}) \tilde{v}^X_{j+1}(z; s_{t+1}) \end{aligned} \tag{18}$$

for $j = 1, ..., J-2$, and

$$\tilde{v}^X_{J-1}(z; s_t) = \tilde{d}^X_{J-1,t}(z) + \beta E_t \frac{\lambda_{t+1}}{\lambda_t} \tilde{v}^X_0(z; s_{t+1}) \tag{19}$$

Solving for this problem recursively leads to the expression for the optimal export price for a typical $j-z$ firm in the form of[6]

$$P_{0,t}^X(z)=\frac{\theta}{\theta-1}\frac{\sum_{j=0}^{J-1}\beta^j\frac{\omega_j}{\omega_0}E_t\alpha_{j,t+j}^X(z)\frac{\lambda_{t+j}}{\lambda_t}\tau_{t+j}\frac{w_{t+j}}{Z_{t+j}z}(P_{t+j}^*)^{\theta}(Q_{t+j})^{\theta}c_{t+j}^*}{\sum_{j=0}^{J-1}\beta^j\frac{\omega_j}{\omega_0}E_t\alpha_{j,t+j}^X(z)\frac{\lambda_{t+j}}{\lambda_t}(P_{t+j}^*)^{\theta-1}(Q_{t+j})^{\theta}c_{t+j}^*} \tag{20}$$

Note that although the optimal export price is similar in many respects to the expression for the optimal domestic price, there are two important differences. First even though the optimal price still depends on the expected future marginal cost, but now this expected marginal cost also depends on the future size of the iceberg cost. Higher current and expected future iceberg costs increase the optimal export price since firms essentially have to use more production labor in order to sell a given unit of output abroad. Second, the optimal export price is also a function of expected current and future probabilities of exporting, which is a function of expected current and future draw of the fixed costs of exporting. These probabilities of exporting serve to modify the effective discount factor. Note that the law of one price may not hold in our setup due the iceberg cost and uncertain probabilities of exporting. The existence of the random fixed cost of exporting thus provides another channel for the deviation of the law of one price and further justifies firms to price to market.

The optimal export price (and the optimal domestic price) above can be solved in terms of optimal real price recursively for each z using the following marginal value recursions

$$0=\alpha_{0,t}^X(z)\frac{\partial\pi_{0,t}^X(z)}{\partial p_{0,t}^X(z)}+\beta E_t\frac{\lambda_{t+1}}{\lambda_t}(1-\alpha_1)m\tilde{v}_1^X(z;s_{t+1})\frac{1}{1+\pi_{t+1}} \tag{21}$$

$$m\tilde{v}_{j,t}^X(z;s_t)=\alpha_{j,t}^X(z)\frac{\partial\pi_{j,t}^X(z)}{\partial p_{j,t}^X(z)}+\beta E_t\frac{\lambda_{t+1}}{\lambda_t}(1-\alpha_{j+1})m\tilde{v}_{j+1}^X(z;s_{t+1})\frac{1}{1+\pi_{t+1}}\quad \text{for}\quad j=1,...,J-2 \tag{22}$$

$$m\tilde{v}_{J-1}^X(z;s_t)=\alpha_{J-1,t}^X(z)\frac{\partial\pi_{J-1,t}^X(z)}{p_{J-1,t}^X(z)} \tag{23}$$

where $m\tilde{v}_{j,t}^X(z;s_t)=\frac{\partial\tilde{v}_j^X(z;s_t)}{\partial p_{j,t}^X(z)}$ and $\frac{1}{1+\pi_{t+1}}=P_{t-1}/P_t$ represents the inverse of home gross inflation. For each $j-z$, we can solve for the real prices $p_{j,t}^X(z)$ using these marginal value recursions. And unlike the domestic branch, we cannot use a representative productivity to solve the problem. The

[6] This expression is also a generalization of the optimal pricing expression exposited in many studies in the New Keynesian Open Macroeconomics literature, e.g. Obtsfeld and Rogoff (1995).

strategy to get around this issue is detailed in the next section. At this point, we also note that in deriving the optimal export price (20) using the marginal value recursions above, we do not neglect the fact that probabilities of exporting $\alpha^X_{j,t}(z)$ are also endogenously determined by the export prices chosen by adjusting firms. The fact that there is no $\frac{\partial \alpha^X_{j,t}(z)}{\partial p^X_{j,t}(z)}$ terms in the marginal value recursions is simply because these terms cancel out given the expression for the probabilities of exporting described below. The interpretation of this is that firms at the margin of exporting cutoffs are indifferent between exporting or not exporting and receive zero export profits.

2.2.2 Proportion of firms entering the export market

Given the export fixed cost draw and the export price, each firm will only export if the export profit is positive. That is, a $j-z$ firm exports only if $\pi^X_{j,t}(z) \geq w_t \xi_t$, where ξ_t is the fixed cost draw the firm must incur should it decides to export. Hence, the probability of exporting (or the proportion of exporting) for firms with relative productivity z in bin j is given by

$$\alpha^X_{j,t}(z) = F(\pi^X_{j,t}(z)/w_t) \tag{24}$$

This condition holds for each z and all $j = 0, ..., J-1$. This proportion of firms exporting is increasing with z since the gross export profit $\pi^X_{j,t}(z)$ is increasing in z. Moreover, $\alpha^X_{j,t}(z)$ is non-increasing in j (the time since the last price adjustment) for a given z - it is decreasing under non-zero inflation.

In relation to a prominent model of endogenous tradeability of Melitz (2003) and Ghironi and Melitz (2005), our current model implies that there are numerous (exporting) cutoffs (one for each $j-z$ pair) rather than just one cutoff. Furthermore, each of these cutoffs will be in terms of fixed-cost cutoff rather than in terms of productivity cutoff. For each $j-z$ pair, firms (with productivity z that last adjusted prices j periods ago) that draw the fixed costs lower than this cutoff will export, while those firms with fixed costs above this cutoff will optimally choose not to export. These cutoffs are increasing in productivity z and non-increasing in the time since the last price adjustment j. As we will see later, this property of the model, which is due to the assumption of heterogenous random fixed cost drawn from a distribution, goes a long way in terms of matching the empirical evidence exposited in figure 1A. Firms with low productivity levels may still find it profitable to export - but compared to firms with relatively higher productivity levels these firms are less likely to export.

2.2.3 Predetermined prices and the price level

Given the optimal real (average) domestic and export prices $\tilde{p}^D_{j,t}$ and $p^X_{0,t}(z)$ The predetermined average domestic and export prices for each z are given by

$$\tilde{p}^D_{j,t} = \frac{1}{1+\pi_t}\tilde{p}^D_{j-1,t-1} \tag{25}$$

$$p^X_{j,t}(z) = \frac{1}{1+\pi_t}p^X_{j-1,t-1}(z) \tag{26}$$

where each equation holds for $j = 1, ..., J-1$. The latter equation above holds for each z. These prices are part of the state vector s_t.

Next, the consumption based price level is given by

$$P_t = \left[\sum\nolimits_{j=0}^{J-1} \omega_j (\tilde{P}^D_{j,t})^{1-\theta} + \sum\nolimits_{j=0}^{J-1} \omega_j \left\{\int_{z_{\min}}^{\infty} [S_t P^{X*}_{j,t}(z)]^{1-\theta} \alpha^{X*}_{j,t}(z) dG(z)\right\}\right]^{1/(1-\theta)}$$

or dividing by the numeraire (price level) P_t, we have

$$1 = \sum\nolimits_{j=0}^{J-1} \omega_j (\tilde{p}^D_{j,t})^{1-\theta} + \sum\nolimits_{j=0}^{J-1} \omega_j \left\{\int_{z_{\min}}^{\infty} [Q_t p^{X*}_{j,t}(z)]^{1-\theta} \alpha^{X*}_{j,t}(z) dG(z)\right\} \tag{27}$$

2.2.4 Aggregation of value functions and dividends

Here we aggregate the profits and value functions across firms. The average domestic profit across all firms is given by $\tilde{d}^D_t = \sum_{j=0}^{J-1} \omega_j \tilde{d}^D_{j,t}$. Since the mass of firms is unity, this expression also represents the total profits of all home firms from their domestic sales. The average export profit across all firms (including non-exporting firms) can be similarly defined as $\tilde{d}^X_t = \sum_{j=0}^{J-1} \omega_j \tilde{d}^X_{j,t}$, where $\tilde{d}^X_{j,t} = \int_{z_{\min}}^{\infty} \tilde{d}^X_{j,t}(z) dG(z)$ represents the average export profit across $j - z$ firms. Hence, the total profits (domestic and export) of all home firms, or the total dividend received by the household, is given by

$$d_t = \tilde{d}^D_t + \tilde{d}^X_t \tag{28}$$

A similar aggregation applies to firms' values. Hence, the value of the portfolio of all firms is

$$v_t = \tilde{v}_t^D + \tilde{v}_t^X \tag{29}$$

where $\tilde{v}_t^D = \sum_{j=0}^{J-1} \omega_j \tilde{v}_{j,t}^D$, $\tilde{v}_t^X = \sum_{j=0}^{J-1} \omega_j \tilde{v}_{j,t}^X$, and $\tilde{v}_{j,t}^X = \int_{z_{\min}}^{\infty} \tilde{v}_{j,t}^X(z) dG(z)$. As expected, expressing the value of all firms above using the definitions of $\tilde{v}_{j,t}^D$ and $\tilde{v}_{j,t}^X(z)$ described previously produces the Euler condition in equation 4

2.3 Aggregate accounting, balanced trade, and the monetary rule

Under financial autarky, the balance of payments equation is

$$c_t = d_t + w_t n_t \tag{30}$$

That is, national income is equal to aggregate consumption expenditure.

Next, the balanced trade condition implied under financial autarky means that the value of export of the home country is equal to the value of export of the foreign country. This expression is given by

$$\left[\sum_{j=1}^{J-1} \omega_j \left\{ \int_{z_{\min}}^{\infty} p_{j,t}^X(z)^{1-\theta} \alpha_{j,t}^X(z) dG(z) \right\}\right] Q_t^{\theta} c_t^* = \left[\sum_{j=1}^{J-1} \omega_j \left\{ \int_{z_{\min}}^{\infty} p_{j,t}^{X*}(z)^{1-\theta} \alpha_{j,t}^{X*}(z) dG(z) \right\}\right] Q_t^{1-\theta} c_t \tag{31}$$

It can be shown that aggregate accounting and balanced trade under financial autarky imply labor market clearing. Hence once we have these two conditions, the labor market clearing condition is redundant.

Finally, to close the model we assume that monetary policy is in the form of nominal money supply growth rule that follows an AR(1) process

$$\triangle M_t = \rho \triangle M_{t-1} + \epsilon_t \tag{32}$$

where ϵ_t is an i.i.d exogenous shock process with mean 0 and standard deviation σ_ε. Hence, we assume no monetary policy spillover across countries.[7]

[7] In this paper we are particularly interested in monetary shocks. Hence, we leave the exogenous processes for the general productivity Z_t and the iceberg cost τ_t unspecified and set them constant.

2.4 Equilibrium definition

The equilibrium is defined as follows. Given the state vector $s_t = [\{p_{j,t-1}\}_{j=0}^{J-2},\ P_{t-1},\ M_{t-1},\ \epsilon_t]$ for the home country and s_t^* for the foreign country, given the monetary rules in both countries, and given the distribution of fixed costs of exporting, (i) households in both countries solve their utility maximization problems; (ii) firms in both countries solve their maximization problems for both domestic and export sales; (iii) the market-clearing conditions hold; and (iv) the balanced trade condition holds. The full list of equations characterizing the equilibrium is summarized in the appendix.

3 Model solution and the steady state

We solve for the model numerically. First, we calculate for the symmetric steady state under zero steady-state inflation. We then log-linearize the model around this steady state to calculate the dynamic behavior of the model. The export branch of the model is not easily solvable since we cannot use the average productivity representation. We choose to simulate the productivity distribution for the export branch by discretizing the distribution. Specifically, for the current benchmark solution, first we condition that the maximum relative productivity to be $z_{\max} = 5$. Starting from $z_{\min} = 1$ to $z_{\max} = 5$, the productivity distribution is discretized with increment 0.25, with each weight derived from the PDF of the Pareto distribution. Hence, this leads to 17 different productivity levels ($z = 1, 1.25, ..., 4.75, 5$). The choice of $z_{\max} = 5$ is motivated by the fact that given the calibrated shape parameter (k) of the Pareto distribution, the weights on all $z > 5$ becomes negligible.[8] Based on these choices of productivity levels to simulate the export branch, a similar approach needs to be made regarding the domestic productivity average in (9). Leaving this domestic average productivity as it is would make the households' expenditure skewed toward domestically-produced goods. Hence, we estimate $\tilde{z}_D$ in (9) by taking the weighted (consumption-based) average of the seventeen productivity levels.

[8] Obviously, the solution would be more precise if we decrease the increment and increase the maximum productivity level.

3.1 Benchmark calibrations

Table 1 displays the benchmark calibrated values for various parameters. We interpret a period in the model as a quarter. The discount rate β is set to 0.99. The parameter governing the risk aversion (σ) is set to 1 (log consumption in utility), which implies an intertemporal elasticity of substitution of 1. The parameter χ is chosen such that households work 20 percent of the allocated time in the steady state. θ=4.33 implies that the mark-up is 30 % in the flexible price equilibrium. As in Ghironi and Melitz (2005), we normalize $z_{\min} = 1$ and choose k such that the standard deviation of the log of domestic US plant sales in the steady state is equal to 1.67 as documented in BEJK (2005). This implies $k = 3.93$. Next, the choice of the iceberg cost τ is in line with that in Obtsfeld and Rogoff (2000).

We use the new evidence on the degree of price stickiness and its distribution in Gopinath and Rigobon (2007) to calibrate the adjustment parameters α_j and ω_j.[9] The parameters in table 1 means that the longest period of price fixity is 9 quarters ($J = 9$) and the probability of adjusting prices after only one quarter (α_1) of price fixity is 8.2 percent. The mean and median of duration of price fixity are 4.22 quarters (12.67 months) and 3 quarters (9 months), respectively. Furthermore the average frequency of price adjustment is 7.9 % per month and the standard deviation of the duration of price fixity is 6.49 months. All these values are in line with those in Gopinath and Rigobon (2007). Figures 2.A and 2.B display the PDF and CDF of the calibrated price duration. Note that in particular, figure 2.B closely matches the evidence on the CDF of price duration in the data as documented by Gopinath and Rigobon (2007).

The choice of the export fixed cost distribution and its parameters is detailed as follows. First, on the choice of the distribution, we use a generalized beta distribution with CDF

$$F(\xi) = \bar{\alpha} + (1 - \bar{\alpha})\Omega(\frac{\xi}{B}; a, b)$$

[9] Gopinath and Rigobon (2007) document that the prices of US imports and exports at the docks are quite sticky in nominal terms (with 12 months duration of price fixity on average), much stickier than the evidence on retail prices in Bils and Klenow (2004). Moreover, heterogenous goods appear to be more sticky than homogenous goods. Since the goods in our model are intermediate goods and firms trade these goods directly, the evidence in Gopinath and Rigobon (2007) is more appropriate for our calibration.

where $\Omega(\frac{\xi}{B}; a, b)$ is the CDF of the beta distribution.[10] The support of this distribution is $\xi \in [0, B]$, where B is the highest possible draw of the fixed cost We introduce the parameter $\bar{\alpha}$ for added flexibility in terms of matching the evidence on proportion of firms exporting in the data. Specifically, $0 \leq \bar{\alpha} \leq 1$ can be interpreted as the proportion of firms receiving zero fixed export costs; when $\bar{\alpha}$ is set to zero, this distribution collapses to the beta distribution. The three other parameters of the distribution ($a > 0$, $b > 0$, and $B \geq 0$) are chosen to match the evidence of exporting firms in BEJK (2005). First, from figure 2A in BEJK (2005) we can obtain the information on the proportion of firms exporting as a function of productivity levels. As we can see from figure 1.A, the proportion of firms exporting overall is increasing with productivity levels and the shape is nearly linear. Since the parameters a and b directly (and B indirectly) affect the shape of the fixed cost distribution, we choose these two parameters so that this proportion of firms exporting (as a function of productivity levels) is linear in the steady state as in figure 1. Finally, B is chosen such that the aggregate proportion of firms exporting is 21% as in BEJK(2005).[11] Figure 2.C displays the probability distribution of the fixed costs of exporting given the chosen parameter values in table 1.

3.2 The steady state

Given the calibrated parameters, the steady state aggregate proportion of firms in the model is 21%. Exporting firms are on average 23% more productive than non-exporting firms. The size (revenue) ratio between exporters and non-exporters in the domestic market is about 1.99. Hence, as shown by BEJK (2005), the model implies that exporters are both more productive and larger in size compared to non-exporters on average. In addition, the share of households' expenditure on imported goods is 23.74%.

[10] The beta distribution is known to be a flexible distribution: the parameters can be chosen such that we can have various shapes of the distribution (convex, concave, decreasing, increasing, etc.). This flexibility is convinient in a sense that we can match the probability of exporting to the evidence in the data by selecting appropriate parameter values (a and b). [11] The choice of $B = 0.52$ (52% of allocated labor hours) may seem high, especially compared to the steady state level of labor effort (0.2). Despite this choice, the actual amount of labor used to pay the fixed cost of exporting is very low since most firms rarely draw this high level of fixed cost and even when firms draw this high fixed cost, they would optimally choose not to export (and not paying the fixed cost). In the model, the aggregate proportion of firms exporting is given by $N_t^X = \sum_{j=0}^{J-1} \omega_j \left\{ \int_{z_{\min}}^{\infty} \alpha_{j,t}^X(z) dG(z) \right\}$ in the home country, and $N_t^{X*} = \sum_{j=0}^{J-1} \omega_j \left\{ \int_{z_{\min}}^{\infty} \alpha_{j,t}^{X*}(z) dG(z) \right\}$ in the foreign country.

Under zero-inflation, the steady-state equilibrium is identical to the flexible-price equilibrium - that is, if inflation is zero, firms would not have to worry about the possibility of not being able to adjust prices since their prices would not be eroded by inflation. It follows that steady state of (average) real domestic price is simply a constant markup over the (average) real marginal cost. In terms of the export branch, this result means that all firms' variables for a given productivity z are the same across bins j (the time since the last price adjustment). For example, the steady state export price of a firm with productivity z in bin 0, a firm that has an opportunity to adjust prices in the current period, is the same as the steady state export price for for other z firms in bins $1, ..., J-1$. The same properties apply to other export variables such as profits, probabilities of exporting, etc.

Next, let's consider the effect of heterogenous random export fixed cost on the export prices in the steady state. As shown in (20), the fixed cost affects the optimal export price through the expected probabilities of exporting α_j^X. Under zero-inflation steady state, these probabilities have no effect on the optimal export prices: since the zero-inflation environment is identical to the flexible-price case, it would be optimal to charge the same price irrespective whether it would export. This is obviously not true out of the steady steady as we will see later in the impulse responses analysis. Hence, like the domestic price, the steady state export price is also a constant markup over marginal cost. On average, the export price is higher than the domestic price due to the existence of the iceberg trade cost - should the iceberg cost is zero, the steady state domestic and export prices would be identical. As expected, both domestic and export prices decrease with the productivity levels.

Last, we consider the steady state proportion of firms exporting as a function of productivity levels. A novel contribution of the paper is that the model can generate a result similar to the data on proportion of firms exporting shown in figure 2A in BEJK (2005). This can be seen in figure 1.B. As in BEJK (2005), the proportion of firms exporting is increasing with the productivity level and it is nearly linear. This result is not possible without the assumption of random fixed export costs drawn from a distribution. The interaction between random heterogenous fixed costs of exporting and heterogenous firms' productivity is crucial in generating this result. Following any exogenous shock to the economy, this line would shift up or down and the slope may change as well.[12] However, it would still be increasing with the productivity level.

4 Accounting for available varieties and the real exchange rate

The purpose of this section are two folds. First, we derive an accounting equation involving the relative availability of goods varieties across countries (up to the first order approximation) and show how this variable evolve in our model. Second, we show how the fluctuations in relative availability of goods varieties may affect the (model-generated) real exchange rate fluctuations. In particular, we show that the magnitude of the effect of endogenous export participation by firms on real exchange rate fluctuations depends positively on the fluctuations of the relative availability of varieties; whether endogenous export decreases or increases the real exchange rate fluctuations, however, depends on the direction of the movements of this relative availability of varieties following shocks. This section thus serves as a building block to understand the impulse responses in the next section. We focus on the responses of these variables following monetary shocks.

In terms of the real exchange rate definition, we follow Ghironi and Melitz (2005) and employ two real exchange-rate definitions: (i) the welfare-based $Q_t = S_t P_t^*/P_t$ defined previously, and (ii) the real exchange rate excluding the varieties effect, $\tilde{Q}_t = S_t \tilde{P}_t^*/\tilde{P}_t = [(1+N_t^{X*})/(1+N_t^X)]^{1/(1-\theta)} Q_t$, where $\tilde{P}_t = [1/(1+N_t^{X*})]^{1/(1-\theta)} P_t$ and $\tilde{P}_t^* = [1/(1+N_t^X)]^{1/(1-\theta)} P_t^*$ represents the average prices of all goods available in the home and foreign country, respectively. Since $\tilde{P}_t$ and $\tilde{P}_t^*$ correspond to the standard constructed CPI price level data more closely, $\tilde{Q}_t$ is a more accurate representation of the real exchange rate in the data. We will however, investigate the effect of endogenous export participation on both Q_t and $\tilde{Q}_t$ below.

Below, we denote the steady state value of variable x_t and its percentage deviation from the steady state as $\bar{x}$ and $\hat{x}_t = \frac{dx_t}{\bar{x}}$, respectively.

Available goods varieties At period t, the number of available goods varieties is $1 + N_{X,t}^*$ and $1 + N_{X,t}$ in the home country and foreign country, respectively. $N_{X,t}$ ($N_{X,t}^*$) is the mass of home (foreign) firms exporting at period t. The availability of these imported varieties depend on the

[12] Since we solve the model using linear approximation around zero steady-state inflation, the slope does not change following shocks. This is because the (static) marginal export profit is zero under zero steady-state inflation. The slope may change if we assume non-zero steady-state inflation.

proportion of firms exporting for all $j-z$ firms as exposited in (24) for firms in the home country. Formally,

$$N_t^X = \sum\nolimits_{j=0}^{J-1} \omega_j \int_{z_{\min}}^{\infty} \alpha_{j,t}^X(z) dG(z)$$
$$N_t^{X*} = \sum\nolimits_{j=0}^{J-1} \omega_j \int_{z_{\min}}^{\infty} \alpha_{j,t}^{X*}(z) dG(z)$$

Without any loss of generality, we assume that the aggregate productivity Z_t (Z_t^*) and the iceberg cost τ_t (τ_t^*) are constant at their steady state levels. Hence, taking the linear approximation of the two expressions above around the steady state, we arrive at the (first-order approximation) expression for the proportion of firms exporting in the home and foreign country given by (see appendix for the derivation)[13]

$$\hat{N}_t^X = \Phi \cdot \ \hat{c}_t^* + [\theta\Phi] \cdot \hat{Q}_t - [\theta\Phi] \cdot \hat{w}_t \tag{33}$$

$$\hat{N}_t^{X*} = \Phi \cdot \ \hat{c}_t - [\theta\Phi] \cdot \hat{Q}_t - [\theta\Phi] \cdot \hat{w}_t^* \tag{34}$$

where $\Phi > 0$ is a constant that depends on the steady state of the variables determining the mass of firms exporting. Note that the two expressions above share the same coefficients due the imposed symmetric steady state equilibrium in the two countries.

Several comments on these two expressions are in order. First, we can break down the effect of a monetary shock on the aggregate mass of firms exporting into several channels. From (33) we can see that: (i) higher foreign aggregate demand increases the number of home firms exporting since it increases the gross export profit ; (ii) an increase in home wage level decreases the number of home firms exporting due to higher cost of producing a given output level and higher fixed cost of exporting; and (iii) the depreciation of real exchange rate increases the mass of firms exporting due to lower prices of home exports relative to foreign exports, which fuels higher foreign demand for home exports and hence increases the gross export profits of home firms (the expenditure-switching channel) . Similar channels affect the mass of foreign firms exporting, except that the depreciation of real exchange rate would decrease $\hat{N}_t^{X*}$. Monetary shocks would affect both $\hat{N}_t^X$ and $\hat{N}_t^{X*}$ through all three channels and hence, affect the number of goods varieties available to home

[13] As this is a general equilibrium model, these equations cannot be interpreted as the solutions of the aggregate masses of firms exporting. They only serve to provide some intuition for analyzing impulse responses in the next section.

and foreign households. An increase (decrease) in $\hat{N}_t^X$ increases (decreases) the number of goods varieties available to foreign consumers, an increase (decrease) in $\hat{N}_t^{X*}$ increases (decreases) the number of goods varieties available to home consumers. Whether the shock increases or decreases the number of available goods varieties in both countries, however, depends on the steady state and hence the structural parameters of the economy. As an illustration, suppose that we have a monetary expansion (an increase in money supply) in the home country. Following this monetary expansion, $\hat{N}_t^X$ may increase or decrease depending on whether the expenditure-switching effect of real exchange rate depreciation dominates the negative effect of higher real wage level in the home country. The model is thus flexible enough to generate any cyclical behavior of exporting firms following monetary shocks should such evidence becomes available.[14] As we will see later in the next section, our benchmark calibration implies that $\hat{N}_t^X < 0$ and $\hat{N}_t^{X*} < 0$ following a monetary expansion at home.[15] Next, notice that individual firms' export prices do not play any role in determining the movements of the aggregate mass of firms exporting up to the first order despite the presence of export prices (through the gross export profit $\pi_{j,t}^X(z)$) in (24). This is due to the fact that we evaluate the variables above around the zero steady-state inflation equilibrium where the marginal export profit for all firms are zero. This means that aggregate shocks to the economy will affect the mass of firms exporting identically across firms irrespective of productivity level and the time since the last price adjustment.

Subtracting (34) from (33) we arrive at the expression governing the relative availability of goods varieties across the two countries

$$\hat{N}_t^X - \hat{N}_t^{X*} = \Phi \cdot \left[-(\hat{c}_t - \hat{c}_t^*) + 2\theta \hat{Q}_t - \theta(\hat{w}_t - \hat{w}_t^*) \right] \tag{35}$$

This is a key equation as the relative availability of goods varieties affect the extent of expenditure-switching effect and relative consumption across countries following a monetary shock, and hence affects the real exchange rate response. $\hat{N}_t^X - \hat{N}_t^{X*} < 0$ means that home consumers enjoy more goods varieties relative to foreign consumers as there are more foreign firms serving the export market relative to home firms. The reverse holds if $\hat{N}_t^X - \hat{N}_t^{X*} > 0$. Following any aggregate

[14] The literature has not provided this evidence on the cyclical behavior of the number of firms exporting (the extensive margin) due to the lack of data at business cycles frequency.

[15] For example, we find that a low enough value of the risk aversion parameter (σ) may generate positive responses of $\hat{N}_t^X$ and $\hat{N}_t^{X*}$ after a home monetary expansion.

shock, the direction of the response of this relative availability of goods varieties depends on the responses of relative aggregate demand, relative real wage level, and the real exchange rate. We note that (35) comes directly from the aggregation of the expression for the probabilities of firms exporting in (24)–hence, this accounting equation holds irrespective of the assumption regarding the financial market completeness. And as showed in the appendix, this expression holds under LCP (local-currency pricing), but with the real exchange rate channel is now interpreted as the channel for changes in firms' revenues from exports due to currency depreciation or appreciation.

Under financial autarky and balanced trade, we can further eliminate relative wage $(\hat{w}_t - \hat{w}_t^*)$ in (35) and express $\hat{N}_t^X - \hat{N}_t^{X*}$ as simply a function of the real exchange rate and relative consumption (see the appendix for the derivation):

$$\hat{N}_t^X - \hat{N}_t^{X*} = \frac{1}{\gamma_N}\hat{Q}_t - \frac{\gamma_C}{\gamma_N} \cdot [\hat{c}_t - \hat{c}_t^*] \tag{36}$$

with the coefficients $\gamma_C > 0$ and $\gamma_N > 0$. Or, using the definition of the empirically relevant real exchange rate $\hat{\tilde{Q}}_t$, we have

$$\hat{N}_t^X - \hat{N}_t^{X*} = \left[\frac{1}{\gamma_N + \frac{\phi^X}{\theta-1}}\right] \cdot \hat{\tilde{Q}}_t - \left[\frac{\gamma_C}{\gamma_N + \frac{\phi^X}{\theta-1}}\right] \cdot [\hat{c}_t - \hat{c}_t^*] \tag{37}$$

where $0 \leq \phi^X = \frac{\bar{N}^X}{1+\bar{N}^X} \leq 1/2$ represents the (symmetric) steady-state share of imported varieties to total goods varieties. So how do monetary shocks affect movements in $\hat{N}_t^X - \hat{N}_t^{X*}$ under financial autarky? And how does the currency-denomination of the prices of the export goods play a role? We argue that following a monetary expansion in the home country, it is likely that $\hat{N}_t^X - \hat{N}_t^{X*} < 0$ under PCP, while the reverse holds under LCP. This result can be understood by considering the relative magnitudes of real exchange rate movements under each currency-denomination assumption following monetary shocks and comparing them with the relative consumption movements.

Consider a monetary expansion (an increase in money supply) in the home country. Following this expansion and since the financial market is incomplete, the relative consumption goes up $(\hat{c}_t - \hat{c}_t^* > 0)$, which *ceteris paribus* negatively affects $\hat{N}_t^X - \hat{N}_t^{X*}$. The home monetary expansion also depreciates the real exchange rate $\hat{\tilde{Q}}_t$, but of different magnitudes across the two currency-denomination assumptions of the export prices. As in standard NOEM models without endogenous export, the real exchange rate depreciation tends to be small under PCP due to the reallocation effect of lower prices of home exports relative to foreign exports, i.e. the expenditure-switching towards home export goods mitigates the relative price movements, and hence tends to decrease the magnitude of real exchange rate depreciation. Under LCP, there is no such mitigating effect, and hence the real exchange rate depreciation tends to be larger. It follows that under LCP, the positive effect of real exchange depreciation is likely to dominate the negative effect of higher relative consumption on the relative availability of goods varieties across countries–hence from (37), there is an increase in the relative availability of varieties ($\hat{N}_t^X - \hat{N}_t^{X*} > 0$). The reverse holds under PCP: the relative availability of varieties tends to decrease ($\hat{N}_t^X - \hat{N}_t^{X*} < 0$) following a monetary expansion in the home country due to lower magnitudes of the real exchange rate depreciation. We note that these movements in $\hat{N}_t^X - \hat{N}_t^{X*}$ will also feed back to endogenously affect movements in $\hat{\tilde{Q}}_t$ and $[\hat{c}_t - \hat{c}_t^*]$, but the above arguments are still valid once the goods market clears. In fact, as we will see later in the impulse response analysis in the next section, these patterns for PCP and LCP tend be the norm across several reasonable parametrizations of the model.

The real exchange rate How do variations in the relative availability of goods varieties affect the real exchange rate dynamics in our general equilibrium setup? The answer is that such variations affect the extent of expenditure-switching effect, which in turns affect the real exchange rate movements.

To be concrete, consider for example a monetary expansion in the home country. In addition, suppose that following this expansion the relative availability of varieties decreases ($\hat{N}_t^X - \hat{N}_t^{X*} < 0$). To be specific, suppose that $\hat{N}_t^{X*} > \hat{N}_t^X > 0$ so that the illustrated case is satisfied. This means that home households now enjoy more goods varieties relative to foreign households since there are more foreign firms exporting relative to home firms. When the set of traded goods is held fixed (the "exogenous" export case as in standard models where $\hat{N}_t^{X*} = \hat{N}_t^X = 0$), households in both countries consume the same number of varieties following this monetary expansion. It follows that the relative consumption increases by more in the endogenous export case compared to the exogenous case. This higher increase in relative consumption thus serves to reduce the extent of the expenditure-switching effect (towards home-produced goods) of home currency depreciation, which in turn endogenously causes the real exchange rate to depreciate by more. Since $\hat{N}_t^X - \hat{N}_t^{X*} < 0$ is likely to occur under PCP in financial autarky based on the previous argument, it follows that the real exchange rate will depreciate by more under the endogenous export case when PCP is assumed. On the contrary, under LCP, where the relative availability of varieties tends to increase following a home monetary expansion ($\hat{N}_t^X - \hat{N}_t^{X*} > 0$), the relative consumption increases by less in the endogenous export case. This smaller increase in relative consumption under LCP in turns creates a channel for expenditure-switching, which endogenously leads to a lower real exchange response compared to the exogenous export case.

We close this section with a note of caution regarding the use of the "expenditure-switching" word above. By expenditure-switching, we do not mean it as the classic expenditure-switching channel in the sense of Mundell-Flemming-Dornbusch. As mentioned previously, it is possibly for the number of home firms (or foreign firms) serving the export market to decrease ($\hat{N}_t^X < 0$) following a home monetary expansion. Hence, in the sense of Mundell-Flemming-Dornbusch, there is now a reduction in the extent of expenditure-switching towards home goods as foreign households are now faced with lower number of imported varieties. So how can the case where $\hat{N}_t^X - \hat{N}_t^{X*} > 0$, but where $\hat{N}_t^X < 0$, be interpreted as creating an expenditure-switching towards home goods? This can be interpreted as a *relative* expenditure-switching effect. That is, *relative* to home consumers, foreign consumers now distribute their consumption expenditure over more goods varieties.

5 Impulse responses

5.1 Benchmark model

In this section we describe the model dynamics following a monetary shock to confirm the predictions in the previous section. To facilitate this, figure 3 displays the impulse responses of some key variables in the model economy under the benchmark calibrations following a 1% temporary increase in the money supply growth in the home country. The persistence parameter ρ is set to 0.9. All variables in figure 3 and subsequent figures are in terms of percentage deviations from their steady state values, except for inflation (in %).

First we discuss the exporting behavior of firms in both countries displayed in the first two columns of figure 3. A monetary expansion at home raises the aggregate consumption and income at home and leads to a nominal exchange rate depreciation on impact. The nominal exchange rate depreciation also leads to the depreciation of the real exchange rate due to the presence of nominal price rigidity. Since prices are denominated in terms of producers' currency, the home terms of trade, defined as the ratio of average price of home export to average price of home import under the same currency, deteriorates following the exchange rate depreciation. The decrease in the prices of home exports relative to foreign exports then leads to higher foreign demand for all home-produced goods (the expenditure switching effect). However, from the perspective of home firms, higher demand from abroad for their products does not translate into higher gross export profits.[16] In fact, the gross export profit decreases for all firms on impact despite higher demand from abroad due to an upward pressure in the marginal cost of production through higher home wage level. From the perspective of (33), higher wage level dominates the extent of expenditure-switching of foreign consumers towards home-produced goods. This in turn leads to a lower number of home firms serving the export market. Turning to the foreign side, the number of foreign firms serving the export market also decreases on impact, albeit of negligible magnitude (about 0.005% on impact). Although foreign wage decreases slightly on impact, the gross export profit decreases because of lower home demand for all foreign-produced goods due to foreign currency appreciation. Hence, from the perspective of (34), the negative effect of expenditure-switching towards home-produced goods dominates the positive effects of higher home aggregate demand and lower foreign wage level on the foreign aggregate mass of firms serving the export market. Comparing the responses of $N_{X,t}$ and $N^*_{X,t}$ in figure 3, we see that the decrease in the number of firms exporting is always larger than the decrease in the number of foreign firms exporting for all periods of monetary stimulation. The resulting exporting behavior of firms in our economy means that home consumers enjoy more goods varieties relative to foreign consumers following a monetary expansion in the home country. Or in other words, there is a temporary decrease in the relative availability of goods varieties across countries ($\hat{N}_t^X - \hat{N}_t^{X*} < 0$).

Turning to the last two columns of figure 3, we see that the increase in home aggregate consumption on impact is almost exclusively fueled by the increase in demand for domestically-produced goods, which again highlights the expenditure switching channel towards home-produced goods of home currency depreciation. Foreign aggregate consumption decreases on impact, albeit quan-

[16] The gross export profit in the figure is the average profit across z for firms in bin $j = 1$. For other bins j, the responses are similar.

titatively negligible. Notice that the expenditure-switching effect of foreign consumers towards home-produced goods is somewhat mitigated in this case despite lower relative prices of home exports (1st row, 4th column). In fact, foreign imported-goods consumption actually slightly decreases on impact, which is fueled by the dominating effect of lower varieties of imported home goods. This latter result highlights the role of endogenous export participation to affect the extent of expenditure-switching effect. When we assume that the set of traded goods is fixed, foreign consumption on imported goods would increase because of lower prices of home exports faced by foreign consumers (due to home currency depreciation). In the case of our present experiment under the endogenous tradeability case, the set of home-produced goods available to be consumed by foreign consumers decreases because a number of home firms optimally choose to cease exporting. Foreign consumers are then forced to redirect some of their expenditure on import goods towards domestically-produced goods because some of previously available home-produced goods become unavailable. This redirection of expenditure serves to affect the response of real exchange rate following a home monetary expansion as we will see shortly in the next figure.

For most variables, peak responses occur after about 3 quarters. As the increase in money stock dissipates and as firms' prices adjust, all variables retreat to their steady state levels.

Endogenous export effect What role do variations in available varieties play in our model? Figures 4.A and 4.B compare the responses of some select variables in our benchmark model to the one in which the available goods varieties are held fixed (the "exogenous" case). Specifically, we shut down the endogenous export participation assumption by holding fixed the proportion of firms exporting in each country and setting them equal to the steady state values of the benchmark model for the both the aggregate proportion of firms exporting and each productivity level (hence, we retain figure 2 at the steady state). Hence, there is no variation in the number of home and foreign firms exporting. The steady state of other variables of this alternative model are similar to the benchmark model with endogenous export participation. We conduct the same experiment of a 1% temporary increase in home money supply growth as before. Note that this experiment of exogenous exporting is isomorphic to the case of an economy with preset non-traded goods.

First notice from figure 4.A that there are important differences in aggregate consumption responses in both countries under the endogenous (solid line) and the exogenous export case (starred line). While foreign aggregate consumption decreases slightly on impact in the endogenous case, it actually increases in the exogenous case. This result is due to fact that there is a decrease in the number of home firms serving the export market ($\hat{N}_{X,t}$) in the endogenous export case, which leads to lower aggregate consumption response on average. Because the number of foreign firms exporting ($\hat{N}_t^{X*}$) also decreases, the response of home aggregate consumption is also smaller in the endogenous than in the exogenous export case. However, the decrease in aggregate foreign consumption ($\hat{c}_t^*$) is larger than the decrease in home consumption ($\hat{c}_t$) because $\hat{N}_{X,t}$ temporarily decreases by more than $\hat{N}_t^{X*}$. Hence, as displayed in figure 4.B, because of the decrease in the relative availability of varieties ($\hat{N}_t^X - \hat{N}_t^{X*} < 0$), the response of relative consumption ($\hat{c}_t - \hat{c}_t^*$) is actually slightly larger in the endogenous export case. This higher response in relative consumption in turn reduces the extent of expenditure-switching toward home goods and causes the real exchange rate, for both Q_t and $\tilde{Q}_t$, to depreciate by more, confirming the argument in the previous section.

5.2 Sensitivity analysis: risk aversion parameter

How large can endogenous export participation increase the real exchange rate fluctuations in our model? Here we present an example where the response of real exchange rate can be particularly large compared to the exogenous tradeability case. Specifically we choose a lower elasticity of intertemporal substitution by setting $\sigma = 5$, while leaving all other parameters the same as in the benchmark calibration.[17]

Figure 5 displays the impulse responses following a 1% increase in home money supply growth. Compared to the benchmark calibration case (figure 3), the dynamics for all variables in figure 5 are qualitatively similar. In particular, the number of firms serving the export market decreases in both countries on impact and in subsequent periods. Moreover, the decrease in the number of firms exporting in the home country decreases by more relative to in the foreign country, which leads to lower relative availability of goods varieties from the standpoint of foreign consumers. Quantitatively however, higher risk aversion leads to larger number of firms exiting the export market in both countries compared to figure 3. On the home country side, this larger decrease

[17] We change the parameters of the cost distribution (B) to maintain the same proportion of firms exporting in the steady state (21%).

in the number of firms exporting is mainly due to higher increase in real wage following a home monetary expansion.

Turning to figures 6.A and 6.B, we see that the relative availability of goods varieties decreases by more compared to the benchmark calibration case. This in turn serves to fuel a higher increase in the relative consumption across countries and a larger real exchange rate depreciation compared to the exogenous case. Specifically, for the more empirically-relevant real exchange rate measure $\tilde{Q}_t$, the peak response of the real exchange rate is more than four times higher in the endogenous case. Hence, the larger is the response of $\hat{N}_t^X - \hat{N}_t^{X*}$ following shocks, the larger is the role of endogenous export participation in affecting the magnitude of the real exchange rate response through the modification of the extent of expenditure-switching.

We close this section by noting that the result above does not depend on the situation where the number of home firms exporting decreases on impact following a home monetary expansion. Figures 9.A and 9.B display a case where the number of firms exporting increases in both countries. Specifically, here we set $\sigma = 0.1$. What matters for the higher response of real exchange rate is the response of *relative* availability of goods varieties. As long as the number of available goods varieties is higher in the home country compared to the varieties in the foreign countries ($\hat{N}_t^X - \hat{N}_t^{X*} < 0$), real exchange rate would depreciate by more in the endogenous tradeability case relative to the exogenous case. This condition certainly holds as can be seen in figure 9.B. Quantitatively, endogenous tradeability does not magnify the real exchange rate response by much in this case due to a lower decrease in relative availability of goods varieties, especially compared to the benchmark calibration case.

5.3 Responses under local-currency pricing (LCP)

Let's turn now to the case when firms are assumed to engage in local-currency pricing (LCP) - hence, export prices are denominated in the currency of the location of the market. We show that under this assumption, endogenous tradeability actually serves to reduce the response of real exchange rate due to higher relative availability of goods varieties following a monetary shock in the home country.

We first briefly discuss the difference of the LCP case compared to the benchmark PCP case (with $\sigma = 1$) under the endogenous tradeability assumption.[18] Figure 7 display the responses of select variables following the same experiment of a 1% temporary increase in money supply growth. Once again, both real and nominal exchange rates depreciate on impact following a monetary expansion at home. The home terms of trade improves on impact due to the LCP assumption. As in the PCP case (figure 5), the number of firms serving the export market decreases in both countries. In the home country, the gross export profits of home firms actually increase on impact due to an appreciation of foreign currency and the fact that home exports are denominated in terms of foreign currency. However, this increase in gross export profits is not enough to dominate the upward pressure in home marginal cost following a home monetary expansion. Hence, the number of exporting home firms decreases. The contrasting increase in gross export profits, however, makes the decrease in the mass of home exporting firms smaller compared to benchmark PCP case. Turning to foreign firms, foreign gross export profits decrease considerably, which is fueled by home currency depreciation. This leads to a considerable decrease (much larger than in the PCP case) in the number of foreign firms exporting. Also note that compared to the PCP case in figure 3, real exchange rate depreciates by more under the LCP assumption as expected.

Next, turning to figures 8.A and 8.B, we see that $\hat{N}_t^X$ decreases by less than $\hat{N}_t^{X*}$, and hence there is an increase in the relative availability of goods varieties on impact ($\hat{N}_t^X - \hat{N}_t^{X*} > 0$). The lack of expenditure-switching effect under LCP leads to a larger exchange rate depreciation that dominates the increase in relative aggregate consumption and relative wage level, which further leads to more goods varieties available for consumption by foreign consumers relative to home consumers. This increase in the relative availability of varieties feeds back to decrease the relative consumption (compared to the exogenous case) and endogenously creates a relative expenditure-switching effect towards home goods, which in turn causes the real exchange rate to depreciate by less in the endogenous export case relative to the exogenous export case.

5.4 Further sensitivity analysis: second moments

For completeness, we quantify the model-generated second moments of several key (Hodrick-Prescott filtered) variables across various alternative parameter values. Specifically, we look at

[18] The PCP and LCP versions of the model share identical steady state.

the standard deviations of the log of the two definitions of real exchange rate ($\hat{Q}_t$ and $\hat{\tilde{Q}}_t$), relative consumption ($\hat{c}_t - \hat{c}_t^*$) and relative availability of imported varieties ($\hat{N}_t^X - \hat{N}_t^{X*}$). The persistence parameter ρ and the standard deviation of exogenous Markov process σ_ε in the money supply rule (32) is set to 0.9 and 1, respectively in each country and we assume that there is no spillover across countries. One can argue that this specification of the money supply rule is not consistent with the data, especially when one is interested in matching the empirical evidence on second moment and cross-correlation of various variables - as shown in various studies, e.g. Baxter (1995) and Chari, et.al. (2002), one needs spillover across countries and capital accumulation to achieve such feats. However, since our purpose is to investigate the role of endogenous export participation in affecting the real exchange rate fluctuations, the current specification is sufficient. Allowing for spillover across countries does not change the main message of the paper.

Table 2 confirms the main predictions of the model. Across different parametrizations of the model, the standard deviation of both measures of real exchange rate are higher under the assumption of endogenous export participation when PCP is assumed. The reverse holds under LCP.[19] As discussed previously, these results stem from the contrasting responses of the relative availability of goods varieties following a monetary shock: under PCP (LCP), more (less) varieties are available in the source country where a monetary expansion is occurring. And as is in the previous impulse response analysis, the magnitude of this endogenous export-participation effect is directly related to the magnitude of the response of relative availability of imported varieties $\hat{N}_t^X - \hat{N}_t^{X*}$ following a shock and hence its volatility. The higher is the volatility of $\hat{N}_t^X - \hat{N}_t^{X*}$, the higher is the effect of endogenous export participation on the volatility of relative consumption and hence, on the real exchange rate volatility. For example, under the benchmark parametrization and PCP case, the standard deviation of $\hat{N}_t^X - \hat{N}_t^{X*}$ is 1.728, which translates into a 54% higher standard deviation of $\hat{\tilde{Q}}_t$ in the endogenous case (0.176) compare to that in the exogenous case (0.114). Under higher risk aversion parameter ($\sigma = 5$), the higher standard deviation of $\hat{N}_t^X - \hat{N}_t^{X*}$ (3.670) yields an almost four times higher standard deviation of $\hat{\tilde{Q}}_t$ in the endogenous case (0.182 compared to 0.050 in the exogenous case). As for relative consumption under PCP, the standard deviation is always higher in the endogenous export case where the volatility of real exchange rate is higher. Once again, the

[19] As expexted, it is still the case that the volatility of real exchange rate is higher under LCP than under PCP across both the endogenous and exogenous export case.

reverse holds under LCP. This pattern holds across different parametrizations of the model.

6 Concluding remarks

This paper develops a two-country model where firms are assumed to face fixed costs of exporting and face infrequent opportunities to adjust prices. The model is able to generate some important trading behavior of firms in the data: (i) the probability of exporting is increasing with productivity level without excluding the possibility of some low-productivity firms exporting, (ii) on average, exporting firms are more productive and have larger size than non-exporting firms, (iii) there are activities of firms entering and exiting the export market.

We derive an accounting equation involving the evolution of relative availability of goods varieties that holds across different assumptions regarding the degree of financial market completeness and the currency-denomination of export goods' prices. Endogenous decision of exporting by firms in the model serves as another channel for the transmission of monetary shocks. Variations in the relative availability of varieties serve to modify the extent of *relative* expenditure-switching, which in turn affects the real exchange rate fluctuations. The model predicts that under financial autarky and balanced trade and when the economy is dominated by monetary shocks, the endogenous-tradeability feature of the model generates higher real exchange rate fluctuations when firms are assumed to engage in producer-currency pricing. Under the local-currency pricing assumption, however, the fluctuations of real exchange rate are dampened. The contrasting response of the changes in relative availability of goods varieties (extensive margin) following monetary shocks provides the exhibited result.

There are several potentially important implications of our results. One obvious implication is in terms of matching the empirical evidence on the real exchange rate volatility along the line of Chari, et. al. (2002). For example, adding the assumption of endogenous export will likely change the degree of risk aversion required to match the standard deviation of real exchange rate in the data. In addition, the combination between endogenous export participation, habit persistence, and non-separable utility seems to be quite promising in terms of solving the relative consumption-real exchange rate correlation puzzle. Recently, there is an outburst of papers studying optimal monetary policy in an open-economy setting. For example, Devereux and Engel (2003) show that the optimal exchange-rate management is different under PCP and LCP due to the lack of expenditure-switching in the latter pricing assumption. Since we show that variations in set of available goods varieties importantly affect the real exchange rate fluctuations through the modification of the extent of expenditure switching, the normative implications of optimal monetary policy in open economy may well be different once we assume that firms can endogenously choose whether to export.

Finally, the model presented here can be readily extended in richer and more realistic settings. The model assumes that firms have no power over the timing and frequency of price adjustments. Adding a state-dependent framework along the line of Dotsey, King, and Wolman (1999) on the top of endogenous export participation seems to be a natural extension. It also seems straightforward to include endogenous currency choice as in Gopinath, Itskhoki, and Rigobon (2007).

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Table 1: Calibrated Parameters (benchmark)

Probability of adjusting prices ($J = 9$)

α_1	α_2	α_3	α_4	α_5	α_6	α_7	α_8	α_9
0.082	0.146	0.215	0.323	0.392	0.426	0.553	0.607	1

Distribution weights of price adjustment ($J = 9$)

ω_0	ω_1	ω_2	ω_3	ω_4	ω_5	ω_6	ω_7	ω_8
0.237	0.218	0.186	0.146	0.099	0.060	0.034	0.015	0.006

Fixed export cost distribution

Proportion of firms receiving zero fixed cost ($\bar{\alpha}$)	0
Shape parameter 1 (a)	0.51
Shape parameter 2 (b)	3.2
The largest possible fixed cost draw (B)	0.52

Productivity distribution (Pareto) parameter

Shape parameter of Pareto distribution (k)	3.93
Minimum relative productivity level ($z_{\min}$)	1

Other parameters

Rate of time preference (β)	0.99
Inverse of labor supply elasticity (η)	0.05
Elasticity of intertemporal substitution (σ)	1
Scaled parameter for steady-state labor (χ)	1.52
Demand elasticity (θ)	4.33
Iceberg cost (τ)	1.3

Table 2: Model-generated second moments of select variables*

	Benchmark			
	PCP		LCP	
std. of	Endog	Exog	Endog	Exog
$\hat{Q}_t$	0.266	0.114	0.965	1.146
$\hat{\tilde{Q}}_t$	0.176	0.114	1.023	1.146
$\hat{c}_t - \hat{c}_t^*$	1.065	1.055	1.109	1.120
$\hat{N}_t^X - \hat{N}_t^{X*}$	1.728	–	1.114	–

	$\sigma = 5$				$\eta = 1$				$\theta = 2$			
	PCP		LCP		PCP		LCP		PCP		LCP	
std. of	Endog	Exog	Endog	Exog	Endog	Exog	Endog	Exog	Endog	Exog	Endog	Exog
$\hat{Q}_t$	0.373	0.050	1.533	1.688	0.292	0.082	1.090	1.264	0.504	0.291	0.983	1.137
$\hat{\tilde{Q}}_t$	0.182	0.050	1.580	1.688	0.179	0.082	1.147	1.264	0.404	0.291	1.032	1.137
$\hat{c}_t - \hat{c}_t^*$	0.476	0.466	0.511	0.514	0.794	0.761	0.914	0.938	1.025	0.992	1.098	1.120
$\hat{N}_t^X - \hat{N}_t^{X*}$	3.670	–	1.297	–	2.354	–	1.154	–	0.578	–	0.283	–

	$\tau = 1^{\dagger}$				Higher degree of price stickiness§				Lower degree of price stickiness††			
	PCP		LCP		PCP		LCP		PCP		LCP	
std. of	Endog	Exog	Endog	Exog	Endog	Exog	Endog	Exog	Endog	Exog	Endog	Exog
$\hat{Q}_t$	0.230	0.073	0.959	1.142	0.371	0.158	1.327	1.592	0.133	0.057	0.492	0.575
$\hat{\tilde{Q}}_t$	0.141	0.073	1.017	1.142	0.245	0.158	1.403	1.592	0.088	0.057	0.523	0.575
$\hat{c}_t - \hat{c}_t^*$	1.005	0.982	1.104	1.129	1.482	1.469	1.542	1.558	0.533	0.528	0.556	0.561
$\hat{N}_t^X - \hat{N}_t^{X*}$	1.716	–	1.124	–	2.405	–	1.488	–	0.864	–	0.589	–

* For each case, the parameter B (the largest possible draw of fixed export cost) is changed appropriately to retain 21% of firms exporting at steady state and country size equal to the benchmark case. "Endog" refers to the model with endogenous export participation, while "Exog" refers to the case where the set of traded sector is exogenously set and held constant. All variables are HP-filtered.

† This case corresponds to zero iceberg trade cost.

§ We retain $J = 9$ and change the probabilites of adjusting prices $\{\alpha_j\}_{j=1}^J$ to [0.014 0.076 0.125 0.193 0.242 0.366 0.423 0.547 1]. This corresponds to mean of duration of price fixity equal to 16.17 months.

†† $J = 9$. $\{\alpha_j\}_{j=1}^J$ = [0.222 0.386 0.425 0.494 0.622 0.786 0.834 0.927 1]. Mean of duration of price fixity = 8.21 months.

Figure 1.A: Proportion of firms exporting vs. productivity (BEJK, figure 2A)

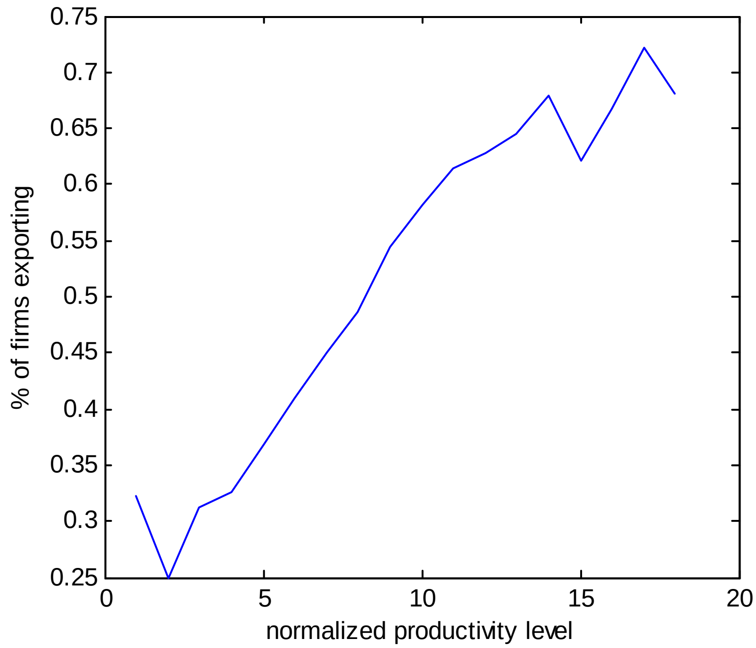


Figure 1.B: Steady state % of firms exporting vs. productivity (model)

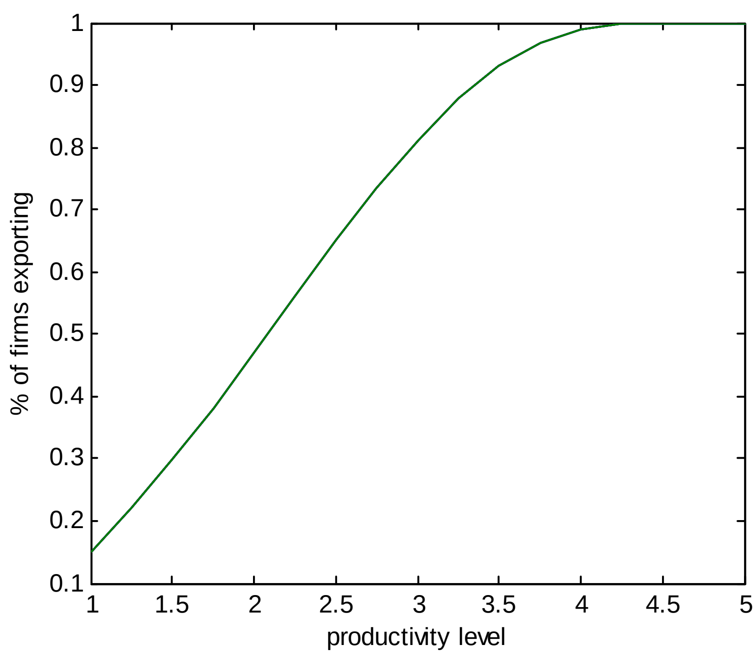


Figure 2.A: Probability distribution (PDF) of price duration

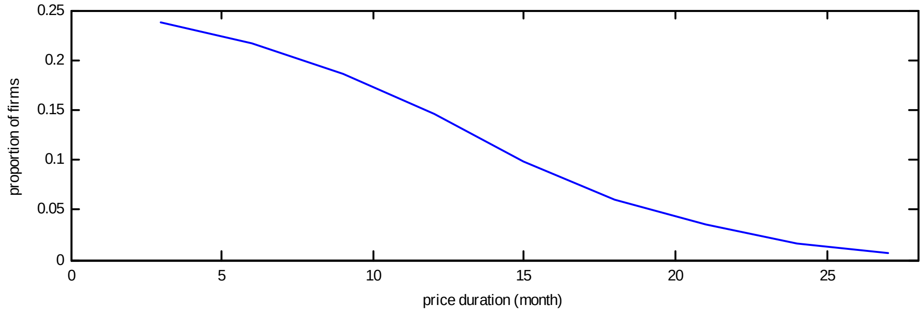


Figure 2.B: CDF of price duration

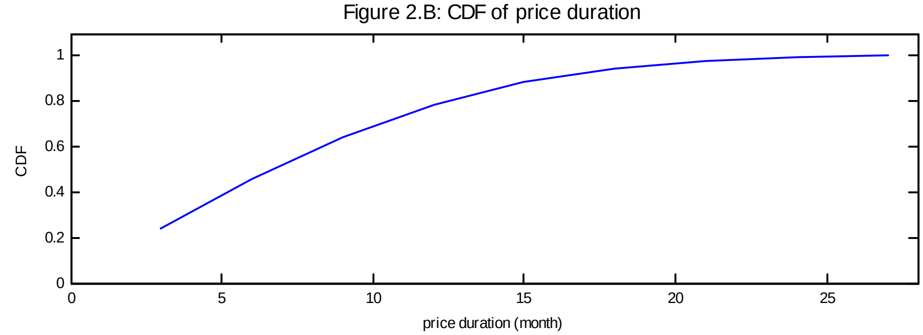
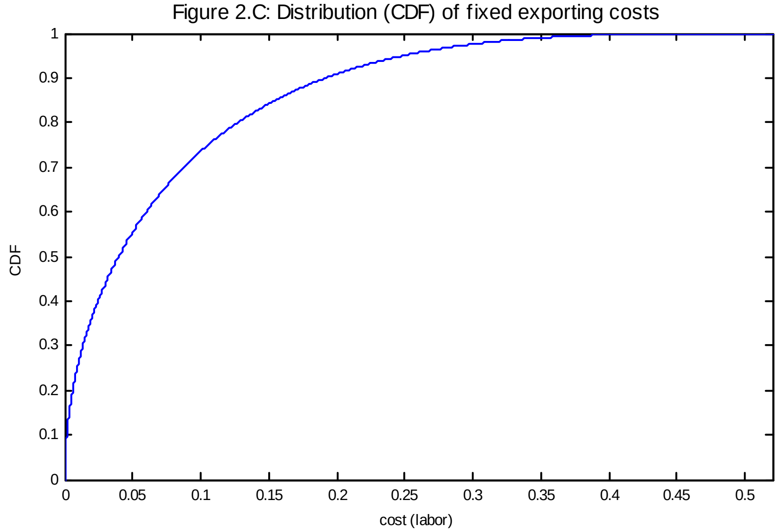


Figure 2.C: Distribution (CDF) of fixed exporting costs



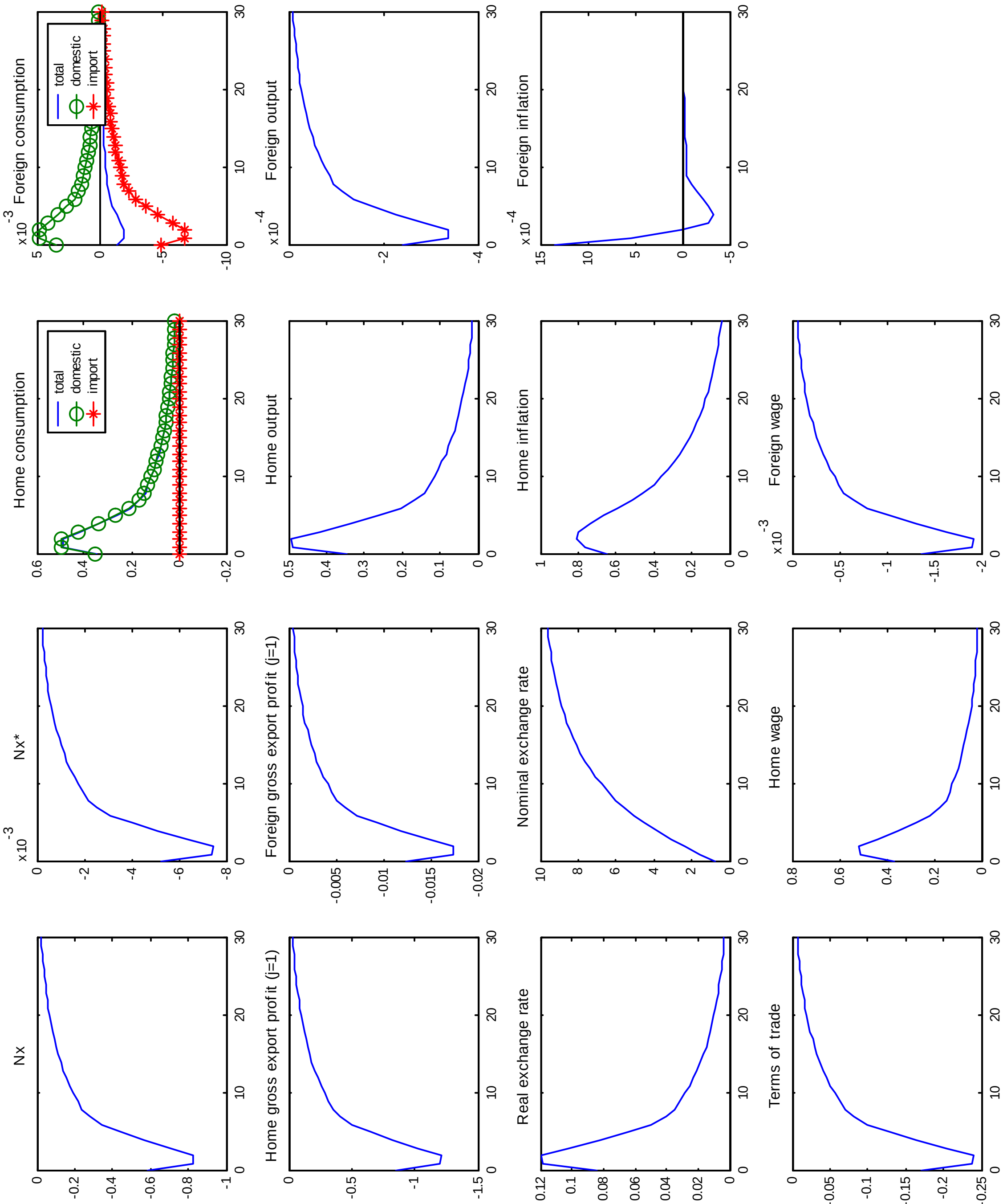


Figure 3: Response to home money shock (benchmark calibration)

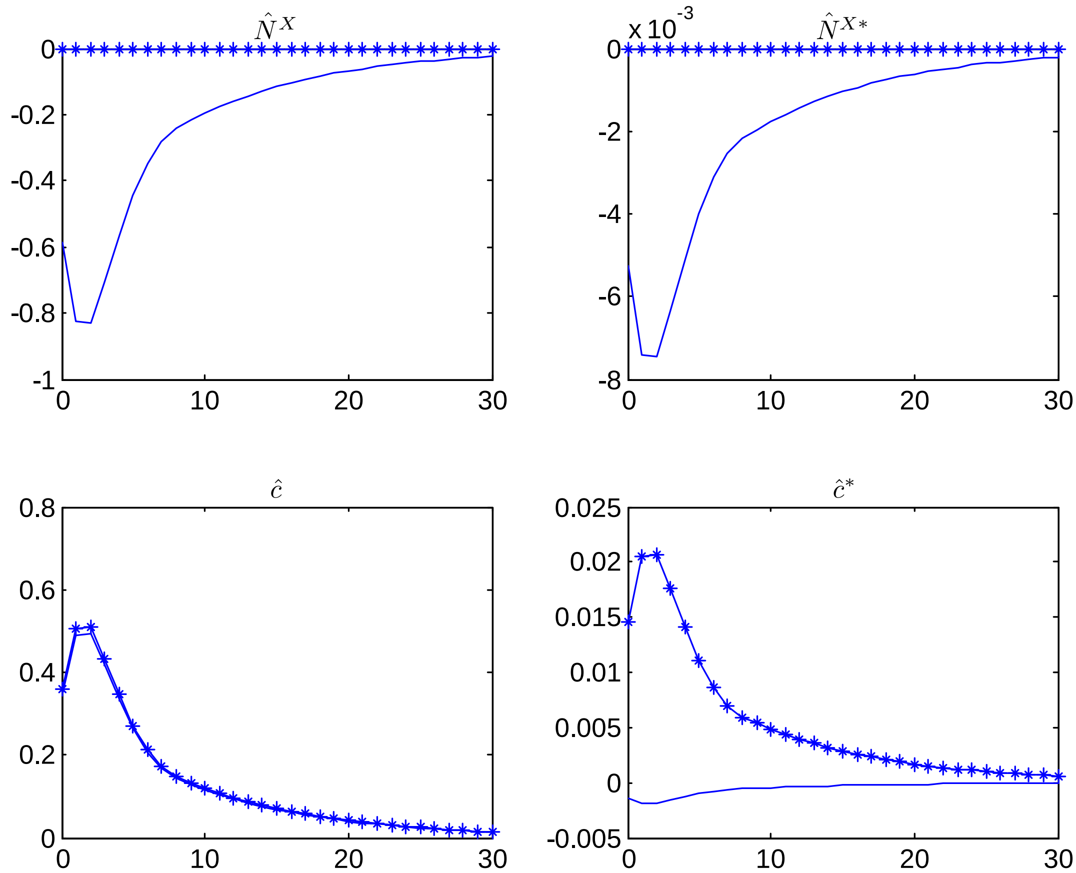


Figure 4.A: Endogenous (—) vs. exogenous (-∗-) exporting under benchmark calibrations (PCP)
(Aggregate proportion of firms exporting and consumption)

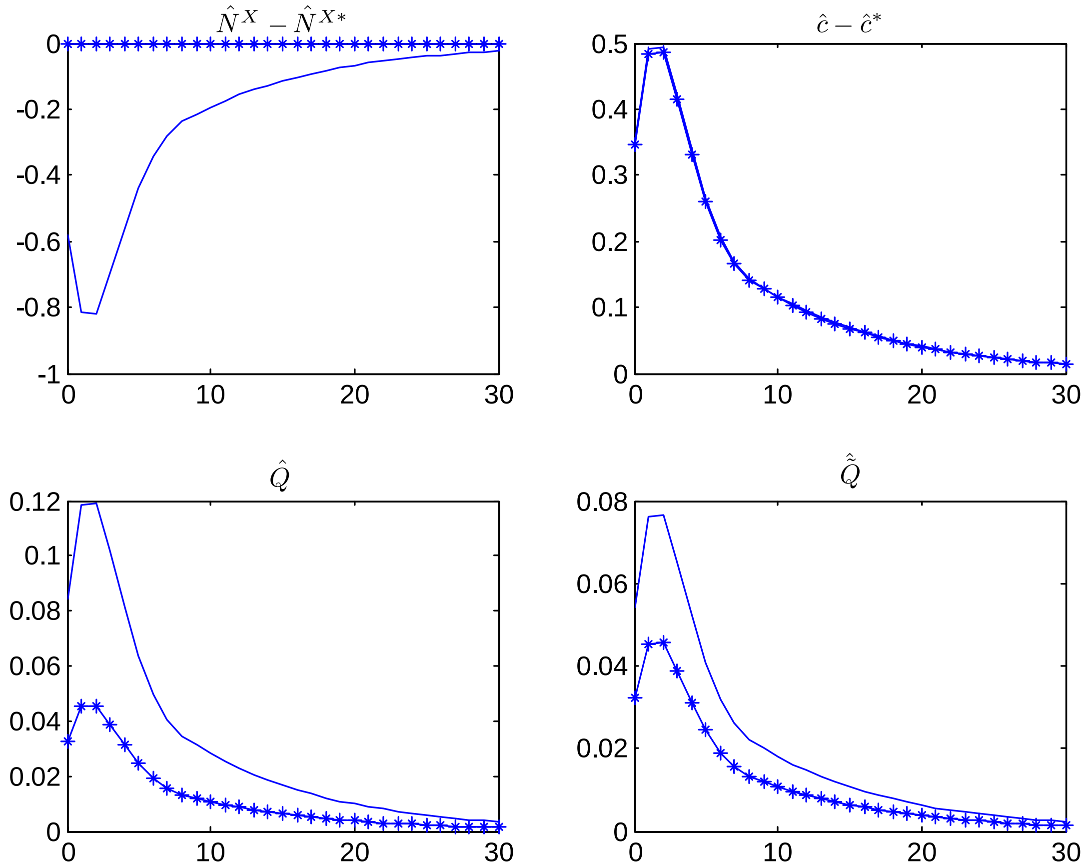


Figure 4.B: Endogenous (—) vs. exogenous (-∗-) exporting under benchmark calibrations (PCP)

(Relative availability of varieties, relative consumption, and the real exchange rate)

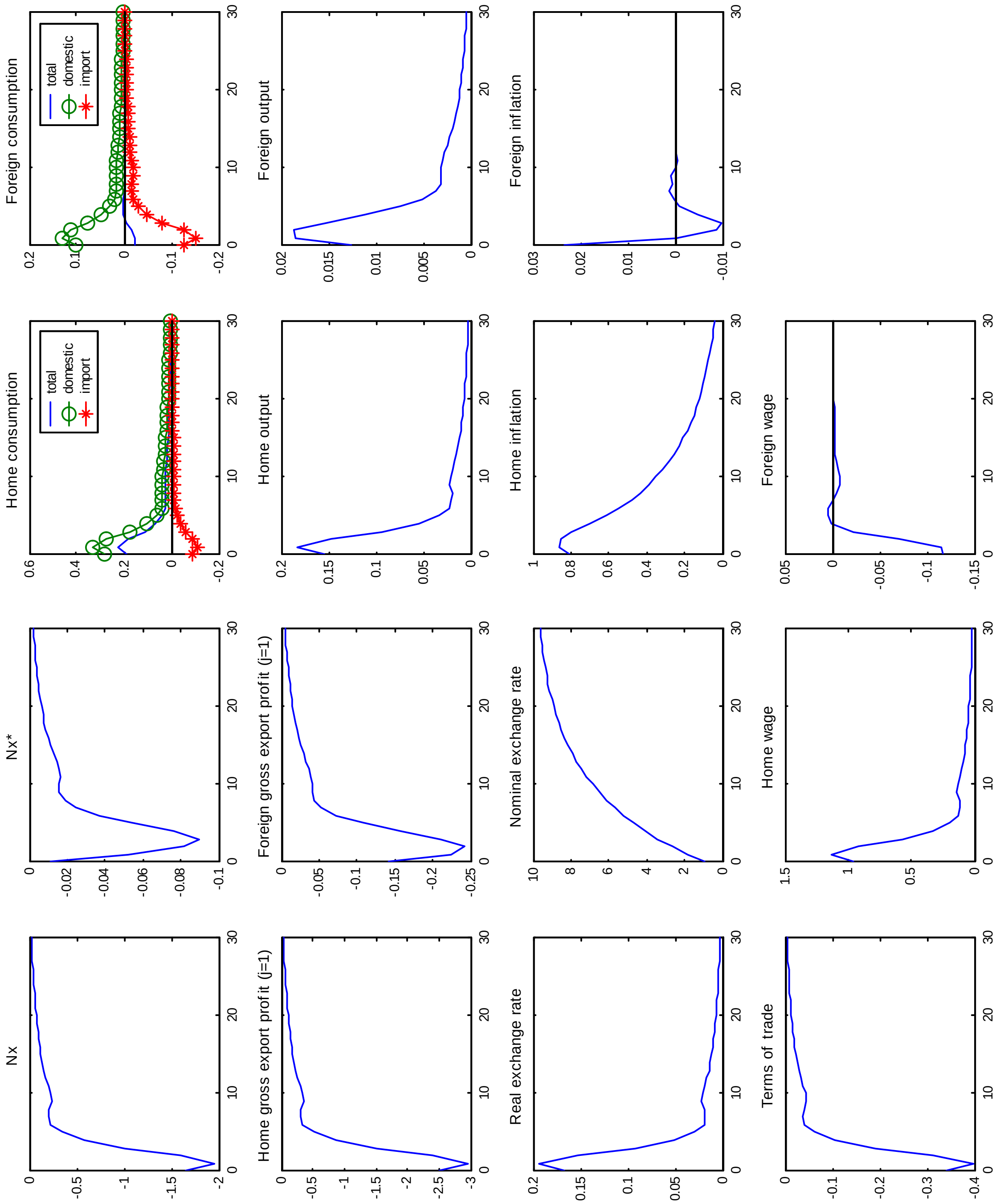


Figure 5: Response to home money shock (higher risk aversion)

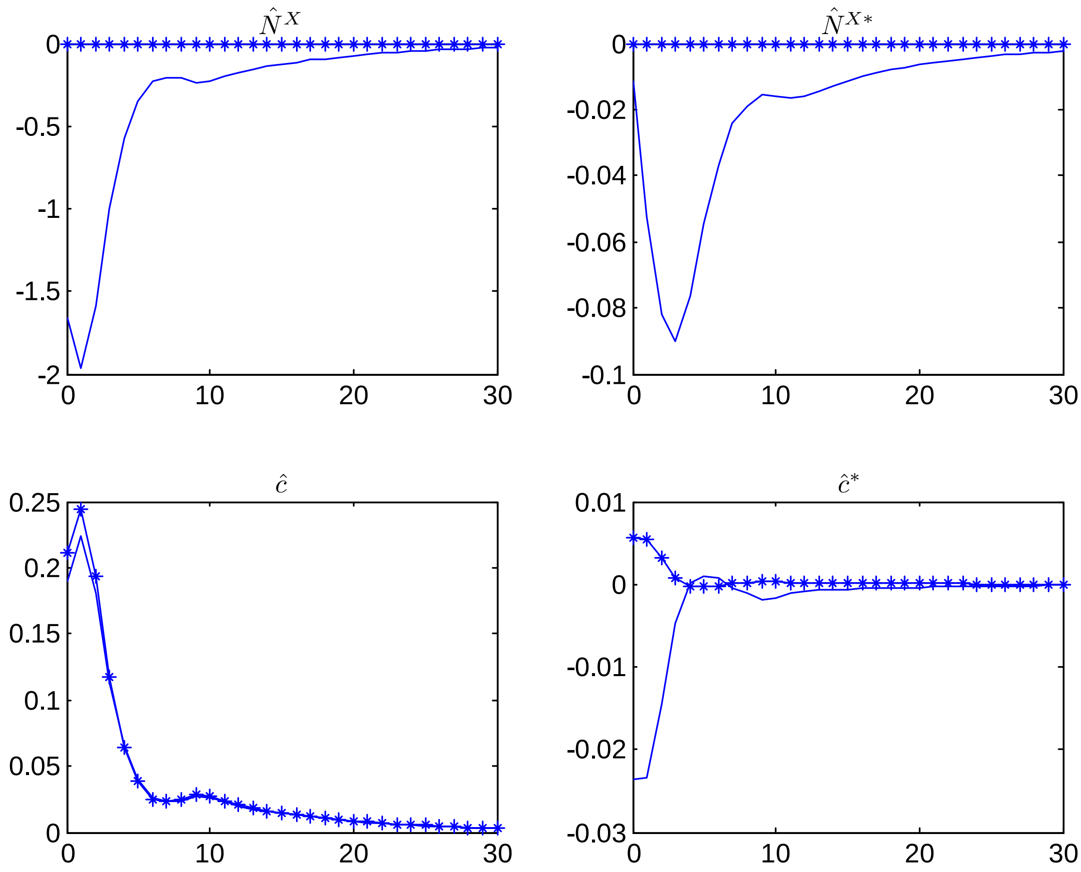


Figure 6.A: Endogenous (—) vs. exogenous (-∗-) exporting under $\sigma = 5$ (PCP)

(Aggregate proportion of firms exporting and consumption)

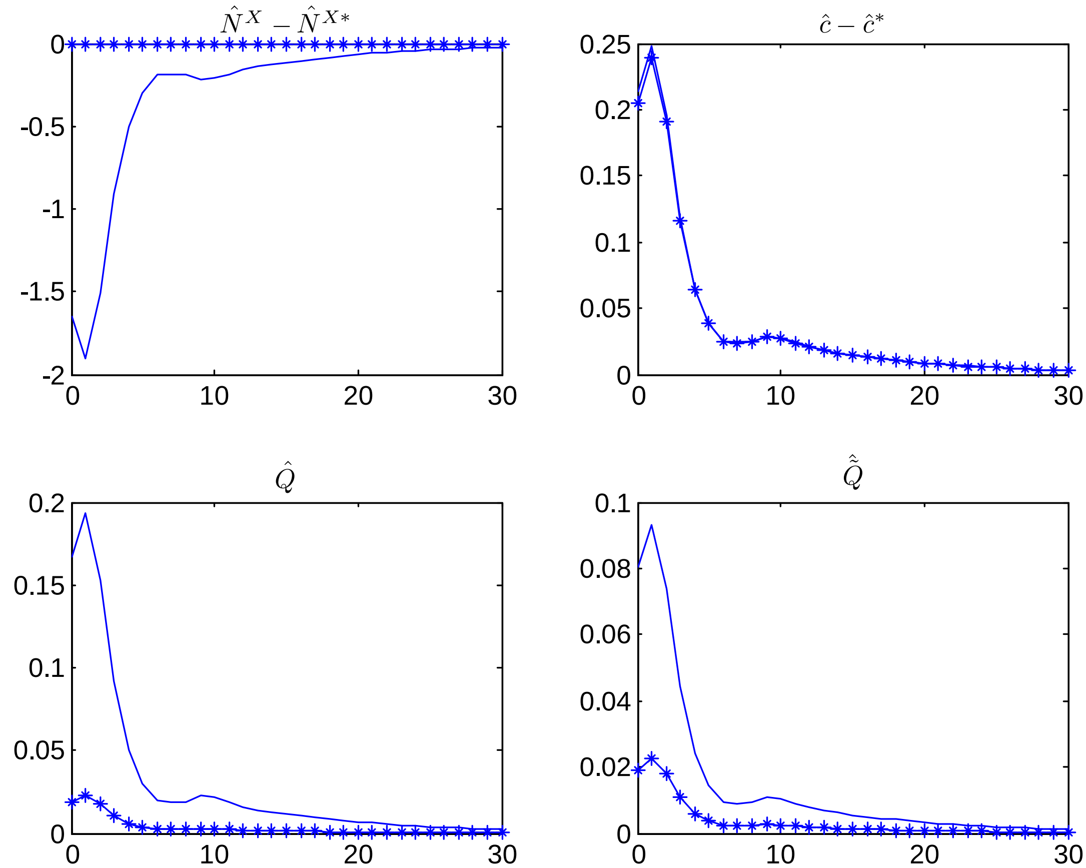


Figure 6.B: Endogenous (—) vs. exogenous (-∗-) exporting under $\sigma = 5$ (PCP)

(Relative availability of varieties, relative consumption, and the real exchange rate)

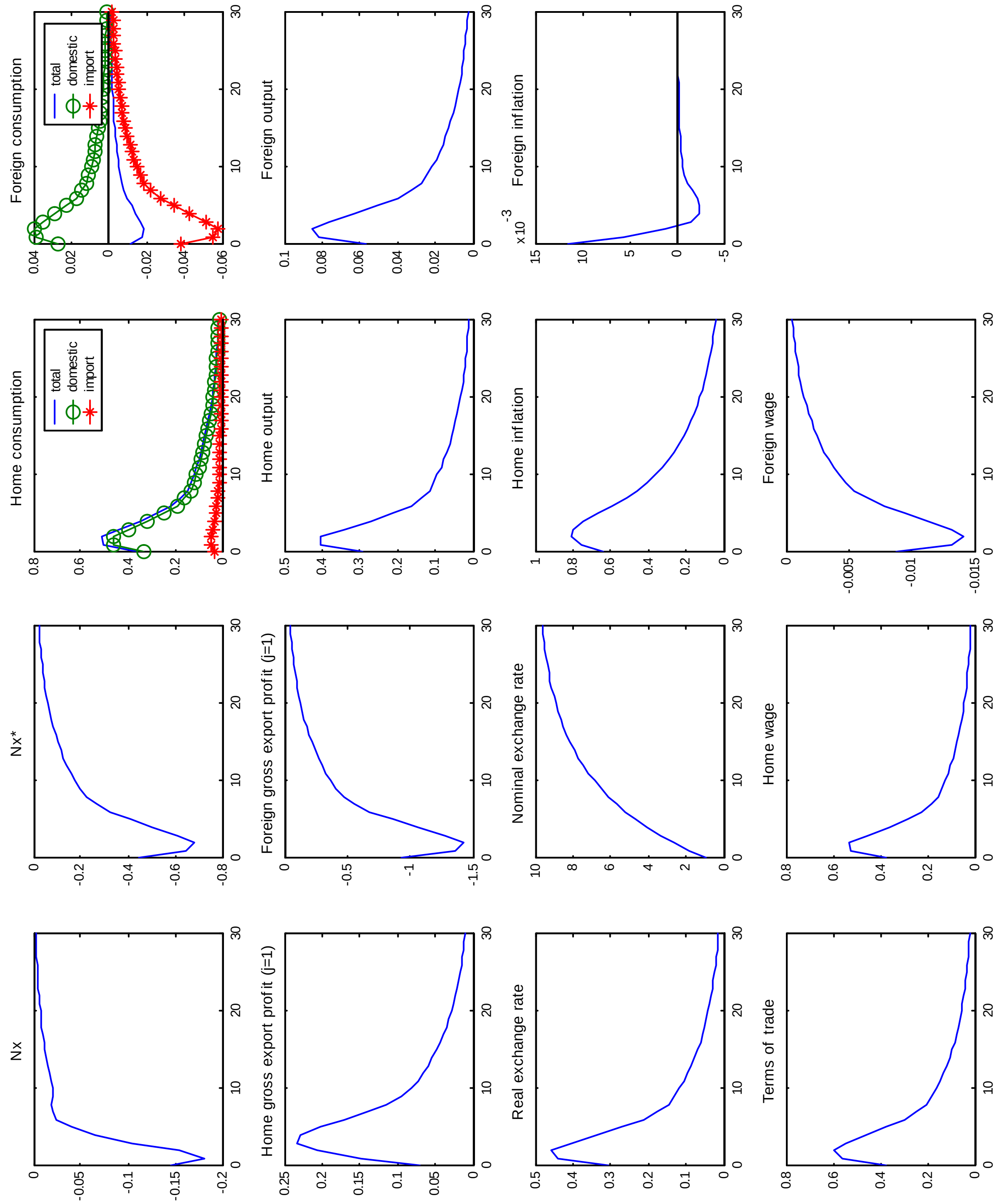


Figure 7: Response to home money shock - LCP

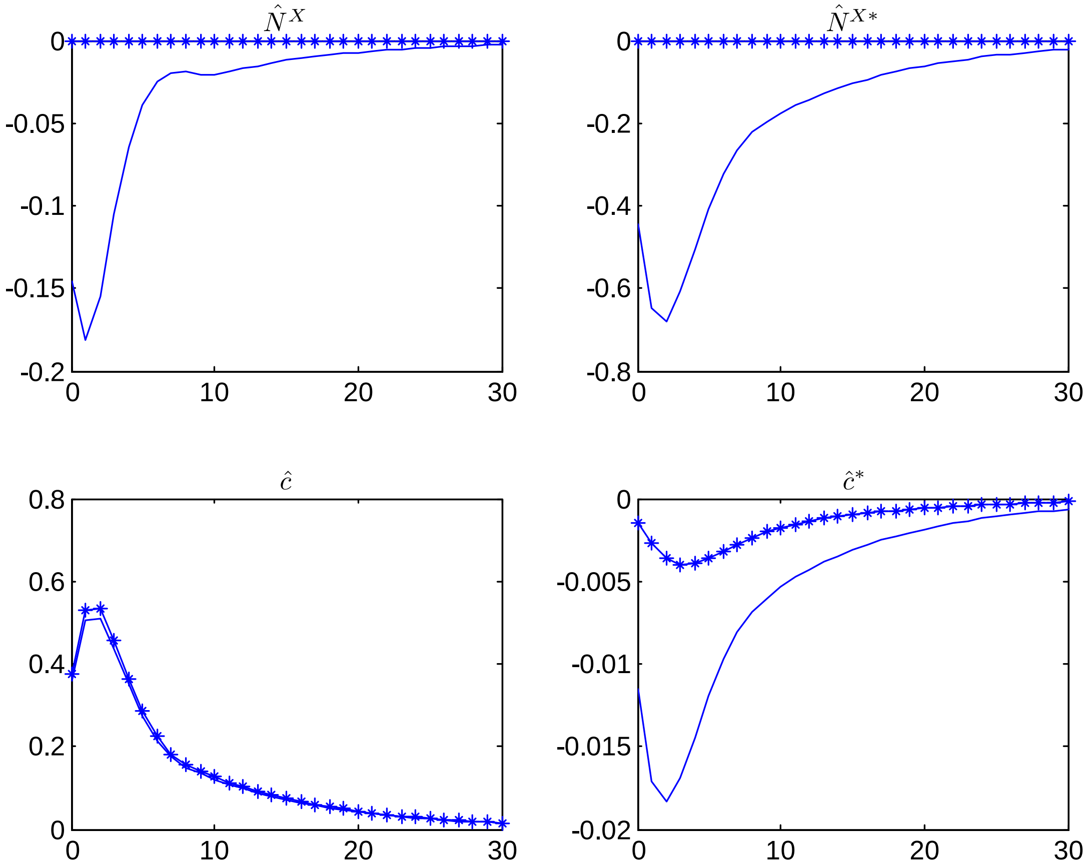


Figure 8.A: Endogenous (—) vs. exogenous (-∗-) exporting under benchmark calibrations (LCP)
(Aggregate proportion of firms exporting and consumption)

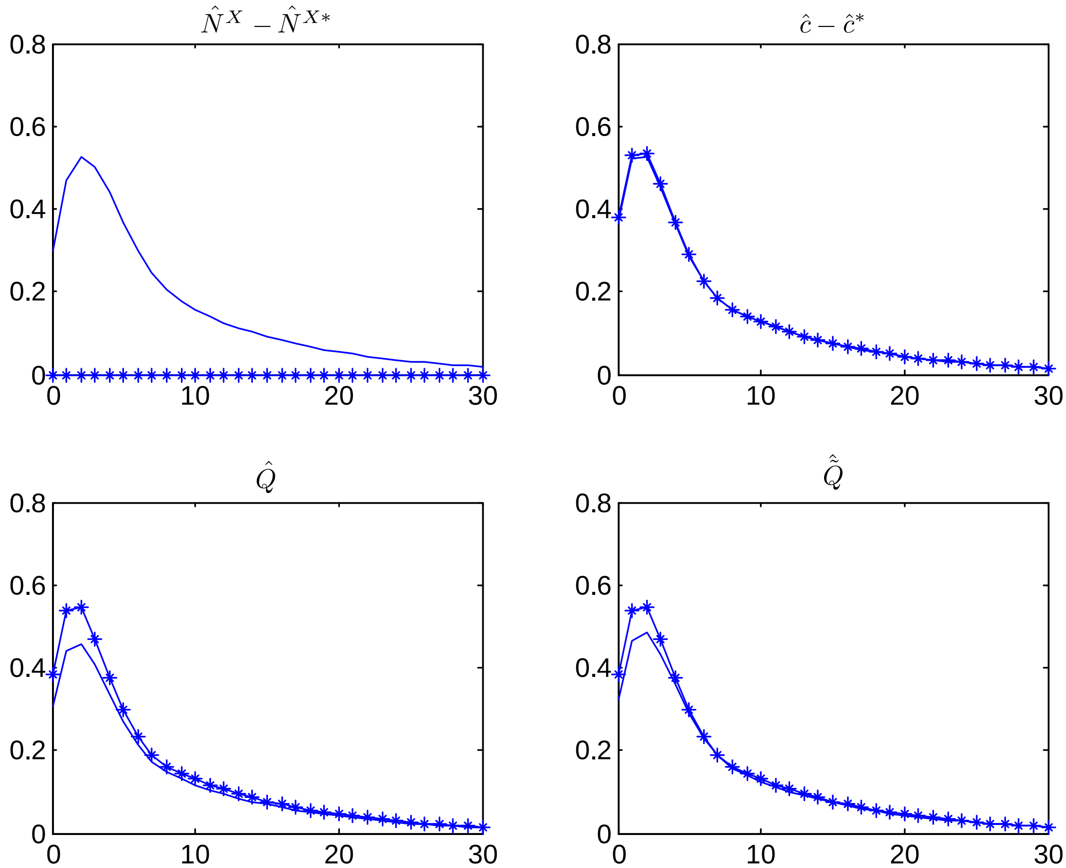


Figure 8.B: Endogenous (—) vs. exogenous (-∗-) exporting under benchmark calibrations (LCP)

(Relative availability of varieties, relative consumption, and the real exchange rate)

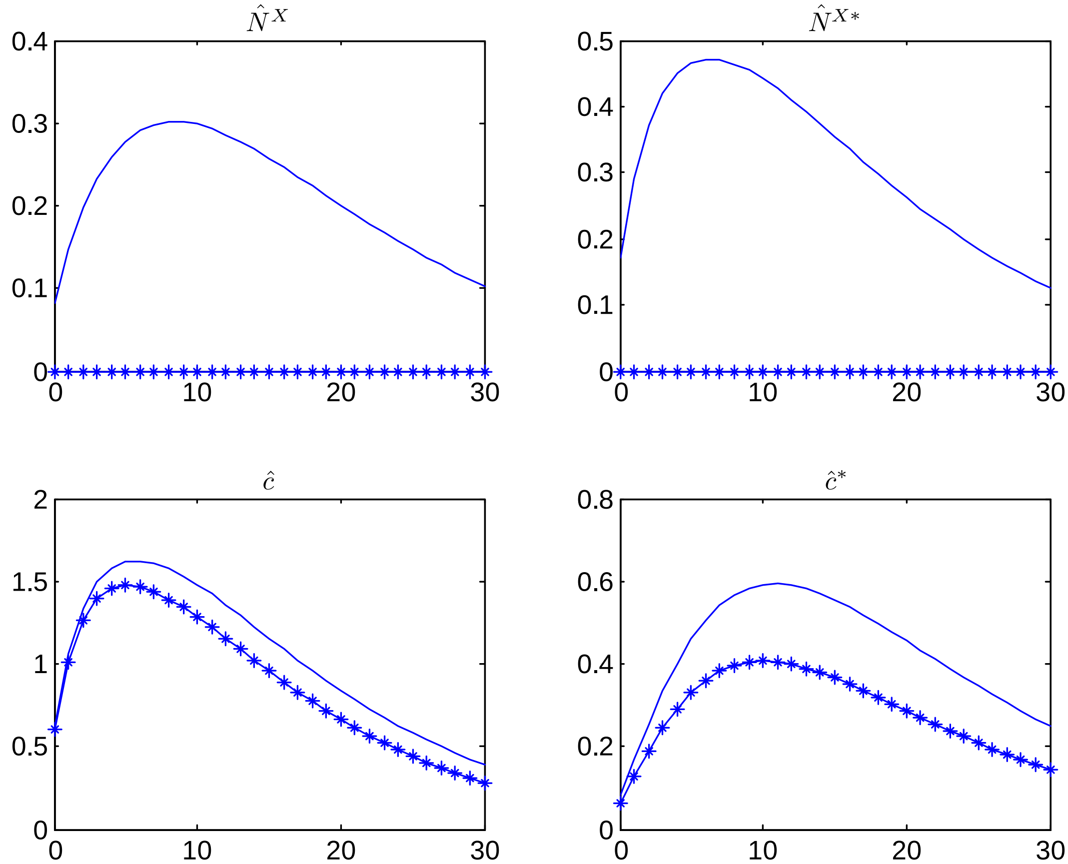


Figure 9.A: Endogenous (—) vs. exogenous (-∗-) exporting under $\sigma = 0.1$ (PCP)

(Aggregate proportion of firms exporting and consumption)

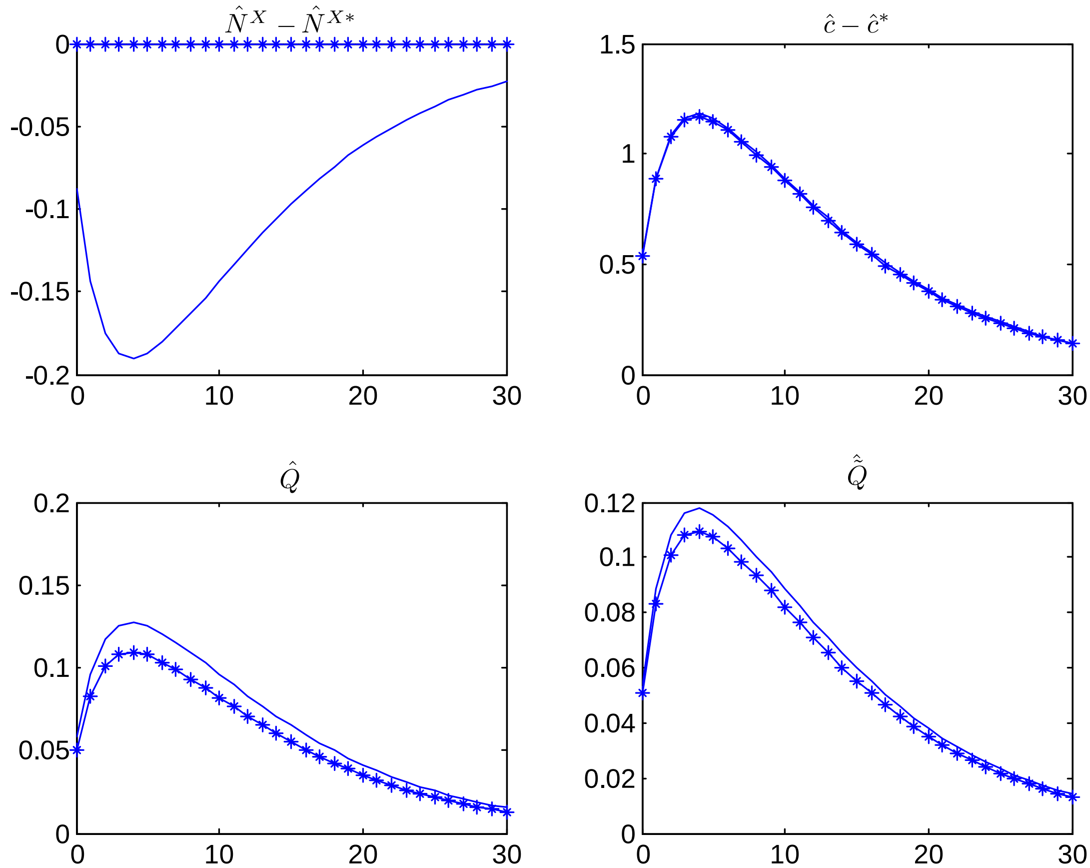


Figure 9.B: Endogenous (—) vs. exogenous (-∗-) exporting under $\sigma = 0.1$ (PCP)

(Relative availability of varieties, relative consumption, and the real exchange rate)

Appendix

A. The LCP (local-currency pricing) version of the model

This appendix spells out the equation of the model when we assume that firms in both countries engage in local-currency pricing. Here, the home (foreign) real export price $p_{j,t}(z)$ $(p^{X*}_{j,t}(z))$ is in terms of foreign (home) consumption unit.

First, the gross export profit under LCP is given by

$$\pi^X_{j,t}(z) = \left[Q_t(p^X_{j,t}(z))^{1-\theta} - (p^X_{j,t}(z))^{-\theta}\tau_t \frac{w_t}{Z_t z}\right] c^*_t \tag{A1}$$

$$\pi^{X*}_{j,t}(z) = \left[Q^{-1}_t(p^{X*}_{j,t}(z))^{1-\theta} - (p^{X*}_{j,t}(z))^{-\theta}\tau^*_t \frac{w^*_t}{Z^*_t z}\right] c_t \tag{A1*}$$

for the firms in home and foreign country, respestively. Since the export prices is in terms of the currency of the destination market, the equations for predetermined export prices become (for $j = 1, ..., J-1$)

$$\tilde{p}^X_{j,t}(z) = \frac{1}{1+\pi^*_t}\tilde{p}^X_{j-1,t-1}(z) \tag{A2}$$

$$\tilde{p}^{X*}_{j,t}(z) = \frac{1}{1+\pi_t}\tilde{p}^{X*}_{j-1,t-1}(z) \tag{A2*}$$

Next, the home marginal export value recursions are given by

$$0 = \alpha^X_{0,t}(z)\frac{\partial \pi^X_{0,t}(z)}{\partial p^X_{0,t}(z)} + \beta E_t \frac{\lambda_{t+1}}{\lambda_t}(1-\alpha_1)m\tilde{v}^X_1(z; s_{t+1})\frac{1}{1+\pi^*_{t+1}}$$

$$m\tilde{v}^X_{j,t}(z; s_t) = \alpha^X_{j,t}(z)\frac{\partial \pi^X_{j,t}(z)}{\partial p^X_{j,t}(z)} + \beta E_t \frac{\lambda_{t+1}}{\lambda_t}(1-\alpha_{j+1})m\tilde{v}^X_{j+1}(z; s_{t+1})\frac{1}{1+\pi^*_{t+1}} \quad \text{for} \quad j = 1, ..., J-2$$

$$m\tilde{v}^X_{J-1}(z; s_t) = \alpha^X_{J-1,t}(z)\frac{\partial \pi^X_{J-1,t}(z)}{p^X_{J-1,t}(z)} \tag{A3}$$

The only difference between the above marginal export value recursions to their PCP counterparts is that the inverse of gross inflation since prices are now denominated in terms of the currency of the destination market. Given this recursion, the optimal (nominal) export price for adjusting firms ($j = 0$) with productivity z is in the form of

$$P^X_{0,t}(z) = \frac{\theta}{\theta-1}\frac{\sum_{j=0}^{J-1}\beta^j \frac{\omega_j}{\omega_0}E_t\alpha^X_{j,t+j}(z)\frac{\lambda_{t+j}}{\lambda_t}\tau_{t+j}\frac{w_{t+j}}{Z_{t+j}z}(P^*_{t+j})^{\theta}c^*_{t+j}}{\sum_{j=0}^{J-1}\beta^j \frac{\omega_j}{\omega_0}E_t\alpha^X_{j,t+j}(z)\frac{\lambda_{t+j}}{\lambda_t}Q_{t+j}(P^*_{t+j})^{\theta-1}c^*_{t+j}}$$

There are some minor differences of this optimal price expression compared to the PCP case, but it still depends on the same set of variables. Similarly, the foreign marginal export value recursions are given by

$$0 = \alpha^{X*}_{0,t}(z)\frac{\partial\pi^{X*}_{0,t}(z)}{\partial p^{X*}_{0,t}(z)} + \beta E_t\frac{\lambda^*_{t+1}}{\lambda^*_t}(1-\alpha_1)m\tilde{v}^{X*}_{1,t+1}(z)\frac{1}{1+\pi_{t+1}}$$

$$m\tilde{v}^{X*}_{j,t}(z) = \alpha^{X*}_{j,t}(z)\frac{\partial\pi^{X*}_{j,t}(z)}{p^{X*}_{j,t}(z)} + \beta E_t\frac{\lambda^*_{t+1}}{\lambda^*_t}(1-\alpha_{j+1})m\tilde{v}^{X*}_{j+1,t+1}(z)\frac{1}{1+\pi_{t+1}} \quad \text{for } j = 1,...,J-2$$

$$m\tilde{v}^{X*}_{J-1,t}(z) = \alpha^{X*}_{J-1,t}(z)\frac{\partial\pi^{X*}_{J-1,t}(z)}{p^{X*}_{J-1,t}(z)} \tag{A3*}$$

Next, the price level equations are now given by

$$1 = \sum_{j=0}^{J-1}\omega_j(\tilde{p}^D_{j,t})^{1-\theta} + \sum_{j=0}^{J-1}\left\{\omega_j\int_{z_{\min}}^{\infty}(\tilde{p}^{X*}_{j,t}(z))^{1-\theta}\alpha^{X*}_{j,t}(z)dG(z)\right\} \tag{A4}$$

$$1 = \sum_{j=0}^{J-1}\omega_j(\tilde{p}^{D*}_{j,t})^{1-\theta} + \sum_{j=0}^{J-1}\left\{\omega_j\int_{z_{\min}}^{\infty}(\tilde{p}^{X}_{j,t}(z))^{1-\theta}\alpha^{X}_{j,t}(z)dG(z)\right\} \tag{A4*}$$

Finally, the balanced trade condition becomes

$$\sum_{j=0}^{J-1}\left\{\omega_j\int_{z_{\min}}^{\infty}(\tilde{p}^{X}_{j,t}(z))^{1-\theta}\alpha^{X}_{j,t}(z)dG(z)\right\}Q_t c^*_t = \sum_{j=0}^{J-1}\left\{\omega_j\int_{z_{\min}}^{\infty}(\tilde{p}^{X*}_{j,t}(z))^{1-\theta}\alpha^{X*}_{j,t}(z)dG(z)\right\}c_t \tag{A5}$$

All other equations are identical to their counterparts in the benchmark PCP case. Despite differences in several equations, both PCP and LCP share the same steady state solution.

B. Derivations of $\hat{N}_t^X$ and $\hat{N}_t^{X*}$

We first derive $\hat{N}_t^X$ under the benchmark PCP assumption. Note that N_t^X depends on $\alpha_{j,t}^X(z)$ for all j and z. Taking the linear approximation of $\alpha_{j,t}^X(z)$ in (24), while making use the definition of gross export profits $\pi_{j,t}^X(z)$, imposing symmetric steady-state, and assuming that the exogenous variables Z_t and τ_t constant at their steady-state levels, we have (for $\forall\ j, z$)

$$\begin{aligned} d\alpha_{j,t}^X(z) &= \phi_j^X(z)\cdot\hat{c}_t^* + [\theta\phi_j^X(z)]\cdot\hat{Q}_t \\ &\quad - \left[\phi_j^X(z)\left\{\frac{\bar{p}_j^X(z)^{-\theta}\bar{\tau}\frac{\bar{w}}{z}\bar{c}}{\bar{\pi}_j^X(z)}+1\right\}\right]\cdot\hat{w}_t \\ &\quad + \left[f(\bar{\pi}_j^X(z)/\bar{w})\frac{\bar{c}}{\bar{w}}\left\{(1-\theta)\bar{p}_j^X(z)^{1-\theta}+\theta\bar{p}_j^X(z)^{-\theta}\bar{\tau}\frac{\bar{w}}{z}\right\}\right]\cdot\hat{p}_{j,t}^X(z) \end{aligned} \tag{B1}$$

where $\phi_j^X(z)\equiv f(\bar{\pi}_j^X(z)/\bar{w})\cdot(\bar{\pi}_j^X(z)/\bar{w})$. Here, $f(.)$ is the PDF of the fixed cost distribution. Next, note that at zero-inflation steady state environment: (i) $\left\{(1-\theta)\bar{p}_j^X(z)^{1-\theta}+\theta\bar{p}_j^X(z)^{-\theta}\bar{\tau}\frac{\bar{w}}{z}\right\}=0$, i.e. the marginal gross export profit is zero; (ii) $\frac{\bar{p}_j^X(z)^{-\theta}\bar{\tau}\frac{\bar{w}}{z}\bar{c}}{\bar{\pi}_j^X(z)}+1=\theta$, making use of the fact that the export price is a mark-up $(\theta/(\theta-1))$ over marginal cost. These two conditions holds for all combinations of $j-z$. Hence, making use of (i) and (ii), (B1) becomes

$$d\alpha_{j,t}^X(z)=\phi_j^X(z)\cdot\hat{c}_t^*+[\theta\phi_j^X(z)]\cdot\hat{Q}_t-[\theta\phi_j^X(z)]\cdot\hat{w}_t \tag{B2}$$

Aggregating (B2) over all j and z, and using the expression for N_t^X and its log-linear approximation (as a function of $\alpha_{j,t}^X(z)$) yield the expression for $\hat{N}_t^X$ as in the paper

$$\hat{N}_t^X=\Phi\cdot\hat{c}_t^*+[\theta\Phi]\cdot\hat{Q}_t-[\theta\Phi]\cdot\hat{w}_t \tag{B3}$$

where $\Phi\equiv\frac{1}{N^X}\left[\sum_{j=0}^{J-1}\omega_j\int_{z_{\min}}^{\infty}\phi_j^X(z)dG(z)\right]>0$.

Using the same steps as above and imposing symmetric steady-state, the expression for $\hat{N}_t^{X*}$ is given by

$$\hat{N}_t^{X*}=\Phi\cdot\ \hat{c}_t-[\theta\Phi]\cdot\hat{Q}_t-[\theta\Phi]\cdot\hat{w}_t^* \tag{B3*}$$

It is easy to verify that similar expressions for $\hat{N}_t^X$ and $\hat{N}_t^{X*}$ as the above hold as well under the LCP assumption. See appendix A for the LCP-version of the model.

C. $\hat{N}_t^X - \hat{N}_t^{X*}$ as a function of relative consumption and real exchange rate only (under financial autarky)

Combining the households' efficiency conditions (2) and (3), we can express the labor supply as

$$n_t = \left[(1/\chi)c_t^{-\sigma}w_t\right]^{1/\eta}$$

Using the balance of payment condition (30) to substitute for n_t above and rearrange yield

$$w_t = \frac{(c_t - d_t)^\delta}{\gamma c_t^{-\rho}} \tag{C1}$$

where $\gamma \equiv (1/\chi)^{1/\eta} > 0$, $\rho \equiv \sigma/(\eta+1) > 0$, and $0 < \delta \equiv \eta/(\eta+1) < 1$. Log-linearizing (C5) around the steady state yields

$$\hat{w}_t = \left[\frac{\delta\bar{c}}{(\bar{c}-\bar{d})} + \rho\right] \cdot \hat{c}_t - \left[\frac{\delta\bar{d}}{(\bar{c}-\bar{d})}\right] \cdot \hat{d}_t \tag{C2}$$

Next we further derive $\hat{d}_t$ using the definition d_t in (28) and individual firms' domestic and export profits. The log-linear expression for the home aggregate domestic profit is thus given by

$$\hat{d}_t^D = \hat{c}_t - (\theta - 1)\hat{w}_t \tag{C3}$$

Note that we have this simple expression since under zero-inflation steady state, the optimal domestic price is simply a constant mark-up over the steady-state marginal cost. Next, the log-linear expression for the export profit for a $j - z$ firm is given by

$$\begin{aligned}\hat{d}_{j,t}^X(z) \quad &= \quad \left[\frac{\bar{\alpha}_j^X(z)\bar{\pi}_j^X(z)}{\bar{d}_j^X(z)}\right] \cdot \hat{c}_t^* + \theta\left[\frac{\bar{\alpha}_j^X(z)\bar{\pi}_j^X(z)}{\bar{d}_j^X(z)}\right] \cdot \hat{Q}_t \\ &\quad - \left[\frac{\bar{\alpha}_j^X(z)\bar{p}_j^X(z)^{-\theta}\bar{\tau}\frac{\bar{w}}{z}\bar{c} + \bar{w}\bar{\Xi}_j(z)}{\bar{d}_j^X(z)}\right] \cdot \hat{w}_t\end{aligned}$$

The above expression does not depend on the log-linear approximation of the probability of exporting and the firm's export price since the marginal export profit is zero under zero-inflation steady state and the firms at the margin (of exporting) receive zero export profits. Aggregating the expression above over $j - z$ and rewriting the coefficient on $\hat{w}_t$ using the export profit definition yield

$$\begin{aligned}\hat{d}_t^X &= \left[\frac{1}{\bar{d}^X}\sum_{j=1}^{J-1}\omega_j\int_{z_{\min}}^{\infty}\bar{\alpha}_j^X(z)\bar{\pi}_j^X(z)dG(z)\right]\cdot\hat{c}_t^* \\ &+\left[\frac{\theta}{\bar{d}^X}\sum_{j=1}^{J-1}\omega_j\int_{z_{\min}}^{\infty}\bar{\alpha}_j^X(z)\bar{\pi}_j^X(z)dG(z)\right]\cdot\hat{Q}_t \\ &-\left[\frac{1}{\bar{d}^X}\sum_{j=1}^{J-1}\omega_j\int_{z_{\min}}^{\infty}\left(\bar{\alpha}_j^X(z)\bar{p}_j^X(z)^{1-\theta}\bar{c}-\bar{d}_j^X(z)\right)dG(z)\right]\cdot\hat{w}_t\end{aligned}\tag{C4}$$

Given (C3) and (C4), the (log-linearized) aggregate profit is

$$\begin{aligned}\hat{d}_t &= \left[\frac{\bar{d}^D}{\bar{d}}\right]\cdot\hat{c}_t+\left[\frac{1}{\bar{d}}\Pi^X\right]\cdot\hat{c}_t^*+\left[\frac{\theta}{\bar{d}}\Pi^X\right]\cdot\hat{Q}_t \\ &+\left[\frac{\bar{d}^X}{\bar{d}}-(\theta-1)\frac{\bar{d}^D}{\bar{d}}-\frac{1}{\bar{d}}REV^X\right]\cdot\hat{w}_t\end{aligned}\tag{C5}$$

where $\Pi^X \equiv \sum_{j=1}^{J-1}\omega_j\int_{z_{\min}}^{\infty}\left\{\bar{\alpha}_j^X(z)\bar{\pi}_j^X(z)\right\}dG(z)$ and $REV^X \equiv \sum_{j=1}^{J-1}\omega_j\int_{z_{\min}}^{\infty}\left\{\bar{\alpha}_j^X(z)\bar{p}_j^X(z)^{1-\theta}\bar{c}\right\}dG(z)$ represent the steady-state aggregate export profit and export revenue, respectively. Substituting for $\hat{d}_t$ in (C2) using (C5) yield the new log-linearized expression for wage

$$\hat{w}_t=\left[\frac{A_1}{B}\right]\cdot\hat{c}_t-\left[\frac{A_2}{B}\right]\cdot\hat{c}_t^*-\left[\frac{\theta A_2}{B}\right]\cdot\hat{Q}_t\tag{C6}$$

where $A_1 \equiv \delta(\bar{c}-\bar{d}^D)+\rho(\bar{c}-\bar{d})>0$, $A_2 \equiv \delta\Pi^X>0$, and $B \equiv (1-\delta)(\bar{c}-\bar{d})>0$. Next, using the same steps as above (and imposing symmetric steady state), foreign wage is given by

$$\hat{w}_t^*=\left[\frac{A_1}{B}\right]\cdot\hat{c}_t^*-\left[\frac{A_2}{B}\right]\cdot\hat{c}_t+\left[\frac{\theta A_2}{B}\right]\cdot\hat{Q}_t\tag{C6*}$$

Given (C6) and (C6*), the relative wage is

$$\hat{w}_t-\hat{w}_t^*=\left[\frac{A_1+A_2}{B}\right]\cdot[\hat{c}_t-\hat{c}_t^*]-2\theta\left[\frac{A_2}{B}\right]\cdot\hat{Q}_t\tag{C7}$$

Finally, substituting for $\hat{w}_t-\hat{w}_t^*$ in equation (35) in the paper using (C7) yield

$$\left[\hat{N}_t^X-\hat{N}_t^{X*}\right]=\frac{1}{\gamma_N}\hat{Q}_t-\frac{\gamma_C}{\gamma_N}\cdot[\hat{c}_t-\hat{c}_t^*]\tag{C8}$$

where $\gamma_C \equiv \frac{1}{2\theta}\frac{(B+\theta A_1+\theta A_2)}{(B+\theta A_2)}>0$ and $\gamma_N \equiv \frac{1}{2\theta\Phi}\frac{B}{(B+\theta A_2)}>0$. Or, using the definition of the empirically relevant real exchange rate $\hat{\hat{Q}}_t$, (C8) can be rewritten as

$$\left[\hat{N}_t^X-\hat{N}_t^{X*}\right]=\left[\frac{1}{\gamma_N+\frac{\phi^X}{\theta-1}}\right]\cdot\hat{\hat{Q}}_t-\left[\frac{\gamma_C}{\gamma_N+\frac{\phi^X}{\theta-1}}\right]\cdot[\hat{c}_t-\hat{c}_t^*]\tag{C9}$$

where $0 \leq \phi^X = \frac{\bar{N}^X}{1+\bar{N}^X} \leq 1/2$ represents the (symmetric) steady-state share of imported varieties to total goods varieties

Using the same steps as the above, it can be shown that (C8) and (C9) also hold under LCP. This is due to the imposed symmetric zero-inflation steady-state, where $\bar{Q} = 1$.

D. Full list of equations and variables

[available upon request]